Honors Physics
Exam II

October 31 to November 4, 2003

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Solutions

Problem 1

A 500 kg satellite orbits the Earth in a circular orbit whose plane contains the Earth's equatorial plane. Point P is on the surface of the Earth and on the equator and on the line connecting the satellite and the center of the Earth. The position of point P on the equator never moves with respect to the Earth.

a. How high above the Earth's surface (in km) is the satellite?
b. What is its total energy in joules?

Solution:

This satellite is in a so-called *geosynchronous* orbit. The period of rotation is 24 hours. The expression for the orbit radius is:

$$r = \left(\frac{GM_e T^2}{4\pi^2} \right)^{1/3}$$

Using known information about the mass and radius of the Earth we find that

$$r = 42{,}298\ km \text{ and } h = r - R = 35{,}900\ km \text{ altitude}$$

The expression for total energy can be written as:

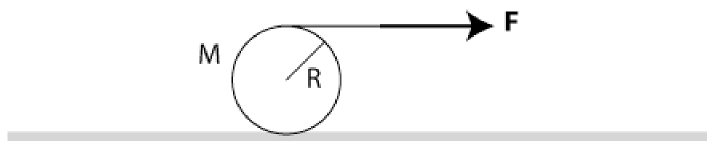
$$E = -\frac{GmM}{2r}$$

and using the information given: $E = -2.36 \times 10^9\ \ J$

Problem 2

A cylinder has mass M and radius R and it rests on a frictionless surface. A string is wrapped around its outside and a constant horizontal force F is applied to the string for a short time period T. *Note: don't worry about the details of the string – it is just there to provide a torque for time T*

a. After time T what is the velocity v of the center of mass of the cylinder?
b. After time T what is the angular velocity ω of the cylinder of the cylinder about its axis?
c. After time T the kinetic energy is equally divided between translational and rotational motion. What is the moment of inertia of the cylinder? Describe the makeup of the cylinder in words .



Solution

The only force acting on the cylinder in the horizontal direction is **F** and during time T is accelerates the center of mass and provides a torque $\tau = FR$ about the center of mass. Assuming that the cylinder has moment of inertia I we have the following expressions for the velocity, angular velocity, translation KE, rotational KE:

$$v = \frac{F}{M}T$$

$$\omega = \frac{\tau}{I}T = \frac{FR}{I}T$$

$$KE_{trans} = \frac{1}{2}M\left(\frac{F}{M}T\right)^2$$

$$KE_{rot} = \frac{1}{2}I\left(\frac{FR}{I}T\right)^2$$

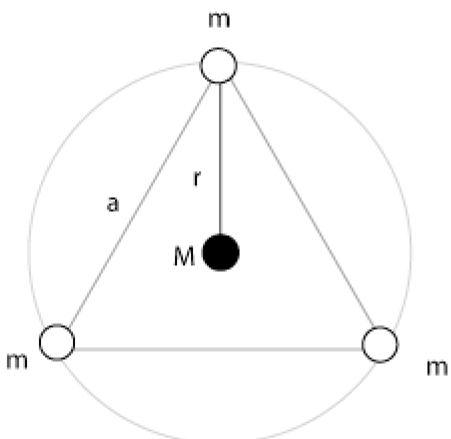
And we see that for the rotational and translational KE to be equal $I=MR^2$

Problem 3

Three planets of equal mass *m* are at the vertices of an equilateral triangle the length of each side being *a*. A massive star of mass *M* is located at the center of mass of the three planets and the star is stationary. The planets move on a circle centered at the star and the distance between them does not change. What is the period of the planets moving around the star? Give your answer in terms of *m, M, a* and *G*

Solution

The drawing shows the situation:



The relationship between r and a: $a = \sqrt{3} \cdot r$

The force on one of the planets due to the other two:

$F_1 = \sqrt{3}\dfrac{Gm^2}{a^2}$ and due to M: $F_2 = 3\dfrac{GmM}{a^2}$

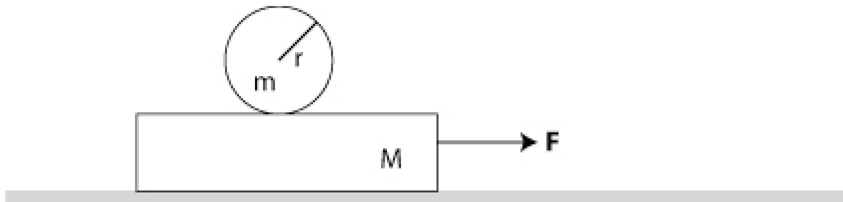
add these and set equal to acceleration times m:

$$\frac{Gm}{a^2}\left(3M + \sqrt{3}m\right) = \frac{mv^2}{r} = \frac{m4\pi^2 a}{\sqrt{3}T^2}$$

and: $T = \sqrt{\dfrac{4\pi^2 a^3}{3G\left(\sqrt{3}M + m\right)}}$

Problem 4

In the figure below the solid ball of mass m and radius r rests on a block of mass M. The block rests on a frictionless surface and the coefficient of friction between the ball and block is μ. A horizontal force **F** is applied to the block. We do not want the ball to slide on the block. What is the maximum value of **F** that satisfies this condition?



Solution

The horizontal force acting on the ball is that due to friction: f. The horizontal forces acting on the block are **F** and f. And friction provides a torque on the ball about its center of mass. So we have:

$$f = ma$$
$$F - f = MA$$
$$\tau = f \cdot r = I\alpha = \frac{2}{5}mr^2\alpha$$

If the ball is to roll on the block without slipping the point on the block and the point on the rim of the ball where ball meets block must be at rest relative to each other so:

$$a + \alpha \cdot r = A$$
$$f \cdot r = \frac{2}{5}mr^2 \cdot \left(\frac{A-a}{r}\right) \Rightarrow f = ma = \frac{2}{5}m(A-a)$$
$$\Rightarrow A = \frac{7}{2}a$$

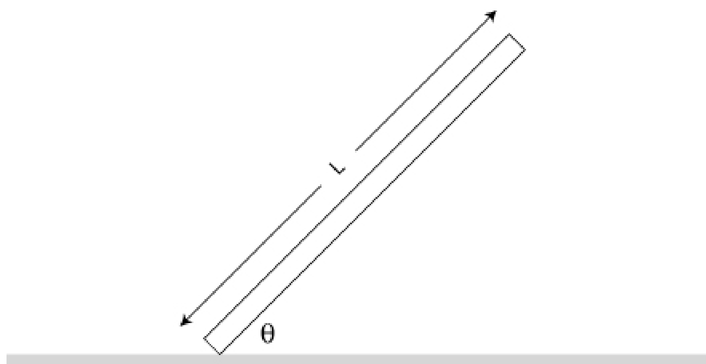
Now we solve for F in terms of a and we know that the greatest value is for a:

$$F = ma + MA = \left(\frac{7}{2}M + m\right)a$$
$$f \le \mu mg \Rightarrow a \le \mu g$$
$$\Rightarrow F \le \left(\frac{7}{2}M + m\right)\mu g$$

Problem 5

The stick (mass M and length L) shown below is held such that it makes angle θ with respect to the floor. The contact between the end of the stick and the floor is frictionless. The stick is released and it falls to the floor.

a. What horizontal distance (if any) does the left end of the stick travel during its fall?
b. With what speed does the right end hit the floor?



Solution

Since there are no forces acting in the horizontal direction the center of the stick falls straight down. So after the stick is flat on the ground the left end has moved to the left by distance d where:

$$d = \frac{L}{2}(1 - cos\,\theta)$$

To answer the second part realize that mechanical energy is conserved. At the instant that the stick hits the ground the potential energy is zero – the energy is all kinetic and before the stick is released it is all potential, so:

$$mg\frac{L}{2}sin\,\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The velocity above is that of the center of mass. When the stick hits the ground the motion is pure rotation about the left end of the stick so:

$$v = \frac{L\omega}{2}$$

$$mg\frac{L}{2}\sin\theta = \frac{m}{2}\left(\frac{L\omega}{2}\right)^2 + \frac{1}{2}\cdot\frac{m(L\omega)^2}{12} = \frac{m(L\omega)^2}{6}$$

$$\Rightarrow \omega = \sqrt{\frac{3g\sin\theta}{L}}$$

and the velocity of the right end is $v_{re} = L\omega = \sqrt{3gL\sin\theta}$