Elastic Scattering – Transforming from LAB to CM

Honors Physics

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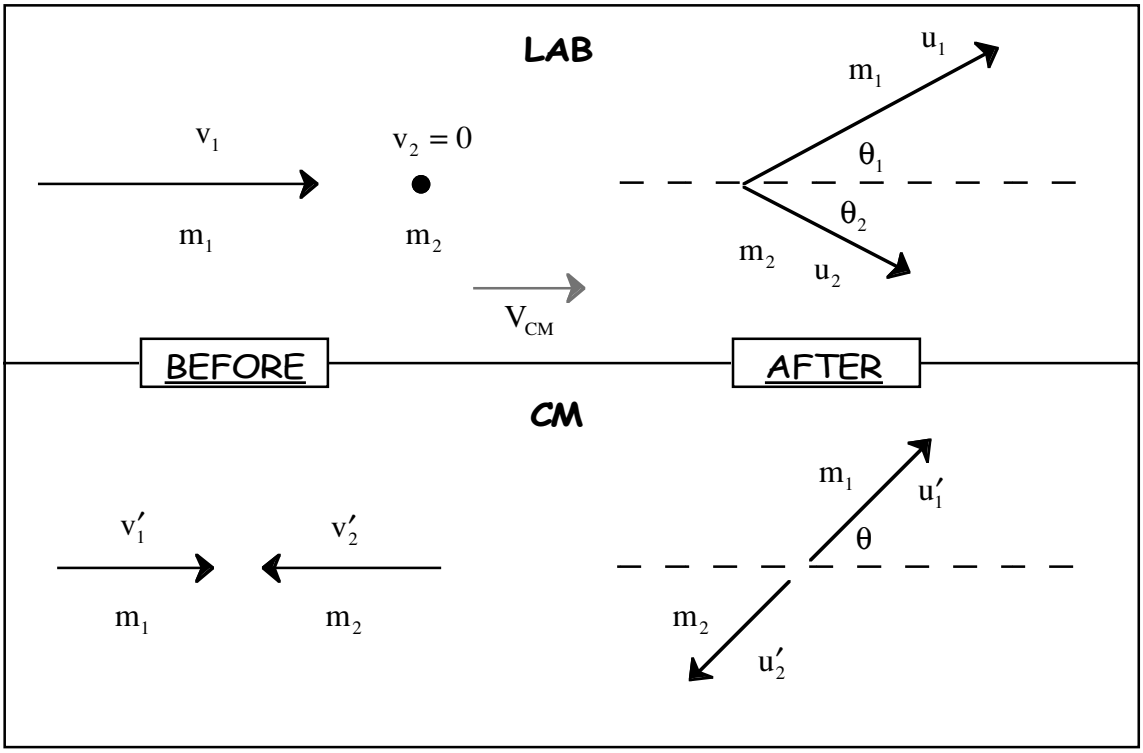
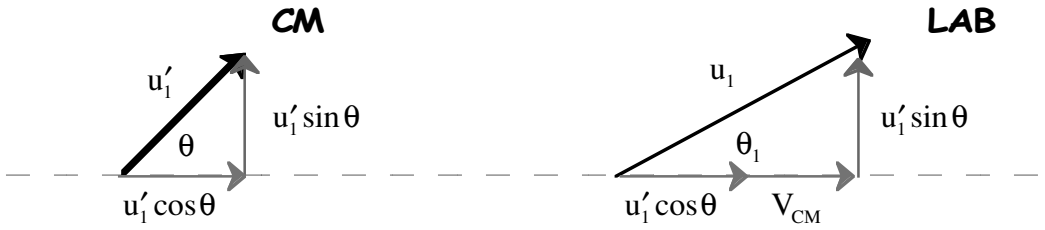
The diagram below shows the elastic scattering of two particles as viewed in the LAB frame and then in the CM (center of mass) frame.





Our notation convention uses the letter v for velocities before the collisions and u for velocities after the collision. We also indicate velocities in the CM with a prime and velocities in the LAB are unprimed. The dashed line in the above indicated the original line along which particle 1 moves in the LAB and it also defines the line along which particles 1 and 2 move in the CM and also the direction of the velocity of the CM, V_{CM}, as seen in the lab. All angles are measured with respect to this line.

In the CM frame the magnitudes of the velocities for particle 1 before and after the collision are unchanged. The same is true for particle 2. This is only the case if the collision is elastic. So we have:

$$u_1' = v_1'$$
$$u_2' = v_2'$$

Note that the LAB velocities can be obtained from the CM velocities by vectorially adding V_{CM} to the CM velocities.

$$V_{CM} = \frac{m_1 v_1}{m_1 + m_2} = \frac{m_1 (v_1' + V_{CM})}{m_1 + m_2}$$

From this you can show that:

$$V_{CM} = \frac{m_1}{m_2} u_1'$$

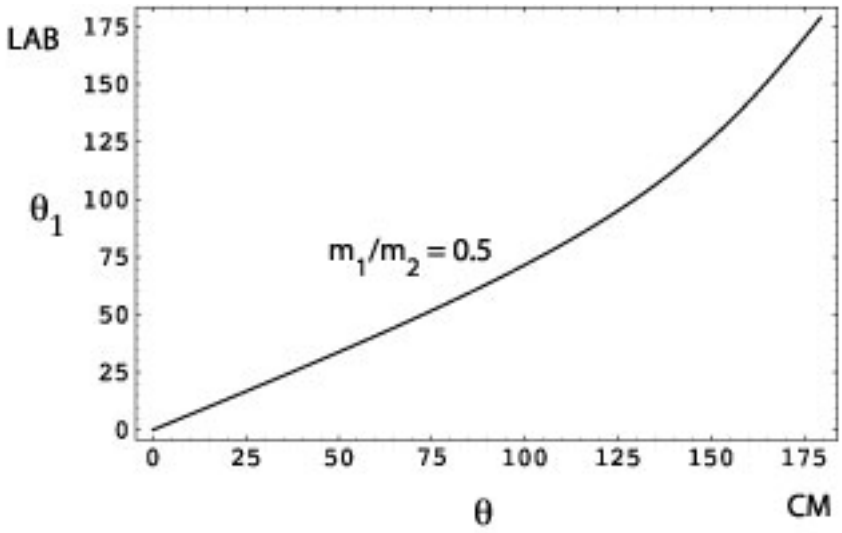
From the above figure you can also relate the scattering angle of particle 1 in the LAB to that in the CM. We note:

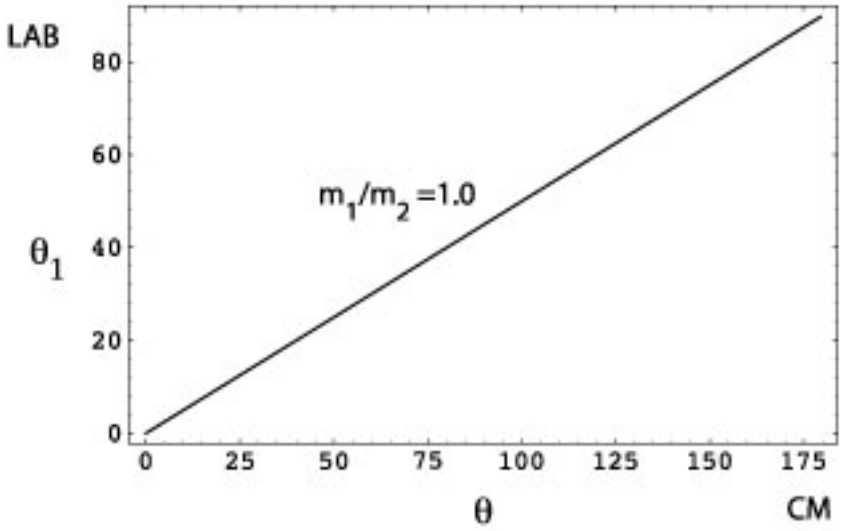
$$\tan\theta_1 = \frac{u_1' \sin\theta_1}{u_1' \cos\theta_1 + V_{CM}} = \frac{\sin\theta_1}{\cos\theta_1 + V_{CM} / u_1'}$$
$$= \frac{\sin\theta_1}{\cos\theta_1 + m_1 / m_2}$$

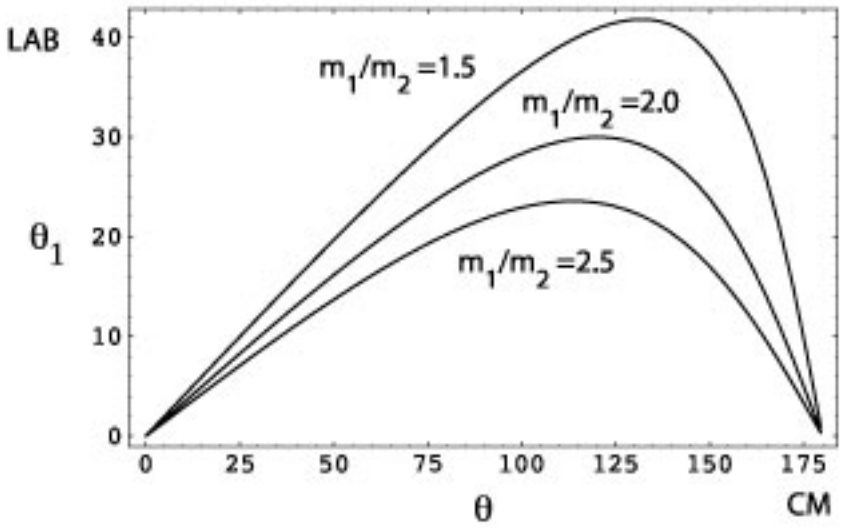
As viewed in the CM, the scattering angle θ can have any value from 0 to 180 degrees but the corresponding LAB angle depends on the ratio of masses. For example, by looking at where the denominator in the above expression vanishes, we see that:

$$m_1 / m_2 < 1 \;\Rightarrow\; \theta_1 < \pi / 2$$
$$m_1 / m_2 = 1 \;\Rightarrow\; \theta_1 \le \pi / 2$$
$$m_1 / m_2 > 1 \;\Rightarrow\; \theta_1 \le \pi$$

The relation between the LAB angle as a function of CM angle is shown below for various mass ratios.


LAB
175
150
125
100
75
50
25
0
θ_1
$m_1/m_2 = 0.5$
0
25
50
75
100
125
150
175
θ
CM


LAB
80
60
40
20
0
θ_1
$m_1/m_2 = 1.0$
0
25
50
75
100
125
150
175
θ
CM


LAB
40
30
20
10
0
θ_1
$m_1/m_2 = 1.5$
$m_1/m_2 = 2.0$
$m_1/m_2 = 2.5$
0
25
50
75
100
125
150
175
θ
CM