Random Errors

Suppose that I make N measurements of a certain quantity x and my measurement errors are random. Then I would report my final answer as as:

$$\mu \pm \sigma$$

average — standard deviation

Where:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2$$

As an example, suppose we measured the period of a pendulum over 5 cycles for a given length and we made 5 such measurements. Below I show the average and standard deviation for the measurement of the period :

5 cycles	**1 cycle**		
10.41	2.082	Average:	**2.0704**
10.28	2.056	Standard Deviation:	**0.0169352**
10.28	2.056		
10.47	2.094		
10.32	2.064		

Using Excel

5 cycles	1 cycle		
10.41	2.082	Average:	**2.0704**
10.28	2.056	Standard Deviation:	**0.0169352**
10.28	2.056		
10.47	2.094		
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So I would report my measurement of the period for this specific length as 2.07 ± 0.02 s.

You can use Excel to find the mean (average) and standard deviation for numbers in a row (say B2 to B6). The functions are:

AVERAGE(B2:B6)

STDEV(B2:B6)

Random Errors and Gaussian Distributions

Suppose a given set of measurements is indeed random and the set is characterized by a certain average or mean: μ and a certain standard deviation: σ. We assume that the distribution of measurements for x will follow a Gaussian distribution given by:

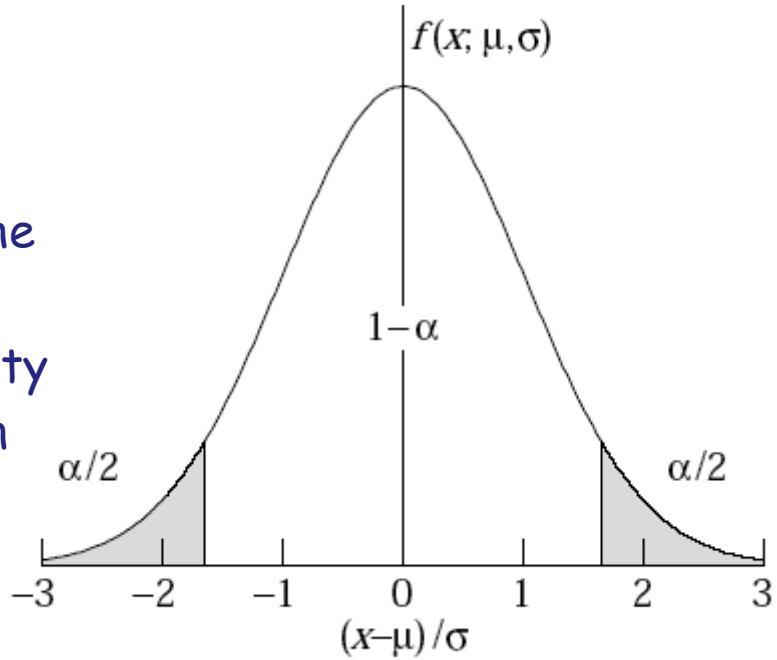
$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-(x-\mu)^2/2\sigma^2)$$

$$-\infty < x < \infty \; ; \qquad -\infty < \mu < \infty \; ; \qquad \sigma > 0$$

The constant in front of the exponential guarantees that the integral of f(x) from minus to plus infinity is 1; that is - the probability of getting some value is 100%. This function allows us to estimate the probability that another measurement of x will deviate from the meanby some specified amount.

Gaussian Distribution

The integral under the Gaussian distribution from ($\mu-\sigma$) to ($\mu+\sigma$) is the probability that another measurement will fall within 1 σ of the mean. According to the table below that is (100-31.7)% = 68.3%. Similarly, the probability that a measurement is within 2 σ of the mean is 95%.



$$1-\alpha = \frac{1}{\sqrt{2\pi}\sigma}\int_{\mu-\delta}^{\mu+\delta} e^{-(x-\mu)^2/2\sigma^2}\,dx$$

Table 31.1: Area of the tails α outside $\pm\delta$ from the mean of a Gaussian distribution.

α (%)	δ	α (%)	δ
31.73	1σ	20	1.28σ
4.55	2σ	10	1.64σ
0.27	3σ	5	1.96σ
6.3×10^{-3}	4σ	1	2.58σ
5.7×10^{-5}	5σ	0.1	3.29σ
2.0×10^{-7}	6σ	0.01	3.89σ