Alex R. Dzierba

Falling Chain

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Imagine holding a chain of length L above the platform of a scale so that it hangs down and just touches the platform. Let it fall freely. The situation is shown in the figure below shortly after the chain is released and the top of the chain has fallen through distance x. The length of chain on the scale is also x. Suppose the mass per length of the chain is λ, then the mass on the scale is λx. The force on the scale, due to the weight of the chain on the scale is $\lambda x g$. But there is an additional force, due to the mass about to hit the platform.

The mass of the length of chain, dx, about to hit the platform is λdx and it carries momentum $dp = \lambda dx\ v$ where v is the instantaneous velocity of the chain. The force on the platform due to dp/dt is $\lambda\ dx/dt\ v = \lambda v^2$. When the chain has fallen through distance x, the velocity is simply given by $v^2 = 2gx$ (remember - this is a free fall). So the contribution to the force from this effect is $2\lambda x g$. So we have the following nifty result: while a chain is freely falling on the platform of a scale the net force on the scale is $\lambda x g + 2\ \lambda x g = 3\ \lambda x g$. *The instantaneous force is 3 times the weight of the chain on the platform at the time. The maximum force exerted is 3 times the weight of the chain.*

I showed this in class with a scale. I also made measurements using the force probe in the ULI package. I used a chain which weighs *2 oz (0.125 lb)* and is *23 inches (1.92 feet)* long. For this chain, $3\lambda g = 0.195\ lb/ft$ so while the chain is falling the dependence of force on time is given by: $f(t) = 3\ \lambda\ g\ (1/2\ g\ t^2)$ or in our case by: $f(t) = 3.125\ t^2\ lb$.

The data are shown below. Data taking was triggered by a force of $0.01\ l$ on the force probe. The sampling rate was 500 Hz. The curve describing the expected time dependence is superposed on the data plot.