Honors Physics P221 Final Exam
Solutions

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Problem 1

A system consists of 0.32 mol of a monatomic ideal gas occupying 2.2 $liters$ at a pressure of 2.4 atm. This defines point A on the P vs V diagram for the gas. The gas is carried through a complete cycle consisting of three parts starting at point A:

1. The gas is heated at a constant pressure until its volume is 4.4 $liter$ at point B.
2. The gas is cooled at a constant volume until is pressure is 1.2 atm at point C
3. The gas undergoes an isothermal compression back to its starting point A.

(a) Compute the temperatures at points A, B and C.

(b) Compute the following quantities: W, Q and ΔE_{int} for each of the three processes above separately and for the entire cycle. Give your answers in $joule$.

Solution to Problem 1

To find the temperatures use:

$$T = \frac{pV}{nR} \tag{1}$$

resulting in: $T_A = T_C = 201\ K$ and $T_B = 402\ K$.

For step (1) $W_1 = p_1 dV = 5.28\ L \cdot atm = 535\ J$ and $Q_1 = \frac{5}{2}nR(T_B - T_A) = 1337\ J$ while $\Delta E_1 = Q_1 - W_1 = 802\ J$.

For step (2) $W_2 = 0\ J$ and $Q_2 = \frac{3}{2}nR(T_C - T_B) = -802\ J$ while $\Delta E_2 = Q_2 - W_2 = -802\ J$.

And for step (3) $W_3 = nRT \ln \frac{V_A}{V_C} = -371\ J$ and $\Delta E_3 = 0\ J$ while $Q_3 = W_3 = -371\ J$.

$W_T = W_1 + W_2 + W_3 = 164\ J$; $Q_T = Q_1 + Q_2 + Q_3 = 164\ J$ and $\Delta E_T = \Delta E_1 + \Delta E_2 + \Delta E_3 = 0\ J$.

Problem 2

The specific heat of carbon dioxide in the temperature range from 300 K to 500 K is approximated by:

$$c(T) = A - \frac{B}{T} + \frac{C}{T^2} \tag{2}$$

where $A = 0.105\ kcal/(kg \cdot K)$, $B = 76.2\ kcal/kg$ and $C = 2.96 \times 10^4\ kcal \cdot K/kg$. Compute the amount of heat required to raise the temperature of 0.250 kg of CO_2 from 300 K to 500 K.

Solution to Problem 2

The heat required:

$$Q = m \int_{300K}^{500K} c(T) dT = m \left[AT - B \ln T - \frac{C}{T} \right]_{300K}^{500K} \tag{3}$$

Substituting values: $Q = 5.38\ kcal$.

Problem 3

A particle of mass $m = 100\ g$ is attached to a spring with constant $k = 500\ N/m$ and the system undergoes simple, undamped, harmonic motion along the x axis with the position of the mass given by $x(t) = A \sin(\omega t + \phi)$. The constants A and ϕ are given by the initial conditions that at $t = 0$ the displacement is $-5\ cm$ and the velocity is $-25\ cm/s$.

(a) Compute A and ϕ.

(b) Compute the numerical value of the total energy of the oscillator, in *joules*, at $t = 4\ s$.

Solution to Problem 3

First calculate $\omega = \sqrt{k/m} = 70.7\ s^{-1}$. At $t = 0$ we have $-0.05\ m = A \sin\phi$ and $-0.25\ m/s = \omega A \cos\phi$ from which we obtain $\phi = 1.5$ and $A = 0.05\ m$.

Since the total energy is a constant with time - evaluate at $t = 0$. But:

$$E = \frac{1}{2}kA^2 \sin^2\phi + \frac{1}{2}m\omega^2 A^2 \cos^2\phi = \frac{1}{2}kA^2 = 0.625\ J \tag{4}$$

Problem 4

A planet of radius R has a hollow core of radius $R/2$ and the total mass of the planet is M. Between $R/2$ and R the density of mass is constant. What is the gravitational force on a small mass m located a distance $3R/4$ from the center of the planet? Give your answer in terms of the gravitational constant G, R, M and m.

Solution to Problem 4

To find the force on m located at $r = 3R/4$ we only need to calculate the mass M' contained inside $r = 3R/4$ and then the force is $GM'm/r^2$. To find M' we calculate $\rho = M/\left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R/2)^3\right]$. Then:

$$M' = \rho\left[\frac{4}{3}\pi(3R/4)^3 - \frac{4}{3}\pi(R/2)^3\right] = \frac{19}{56}M \tag{5}$$

and from this the force is:

$$F = \frac{38}{63}\frac{GMm}{R^2} \tag{6}$$

Problem 5

A rock is suspended from a spring scale and the scale reports the weight of the rock as w_o. Keeping the rock attached to the scale, it is now completely submerged in a liquid of density ρ and now the scale reports the weight as w. In terms of the variables given here, what is the density of the rock?

Solution to Problem 5

Call the density of the rock ρ_r and the weight of the rock is $w_0 = \rho_r V g$ where V is the volume of the rock. When it is completely submerged the buoyant force of the water is $\rho_W V g$ where ρ_W is the density of water. The scale reports $w = w_0 - \rho_W V g$ and from all this:

$$\rho_r = \frac{w_0}{w_0 - w}\rho_W \tag{7}$$