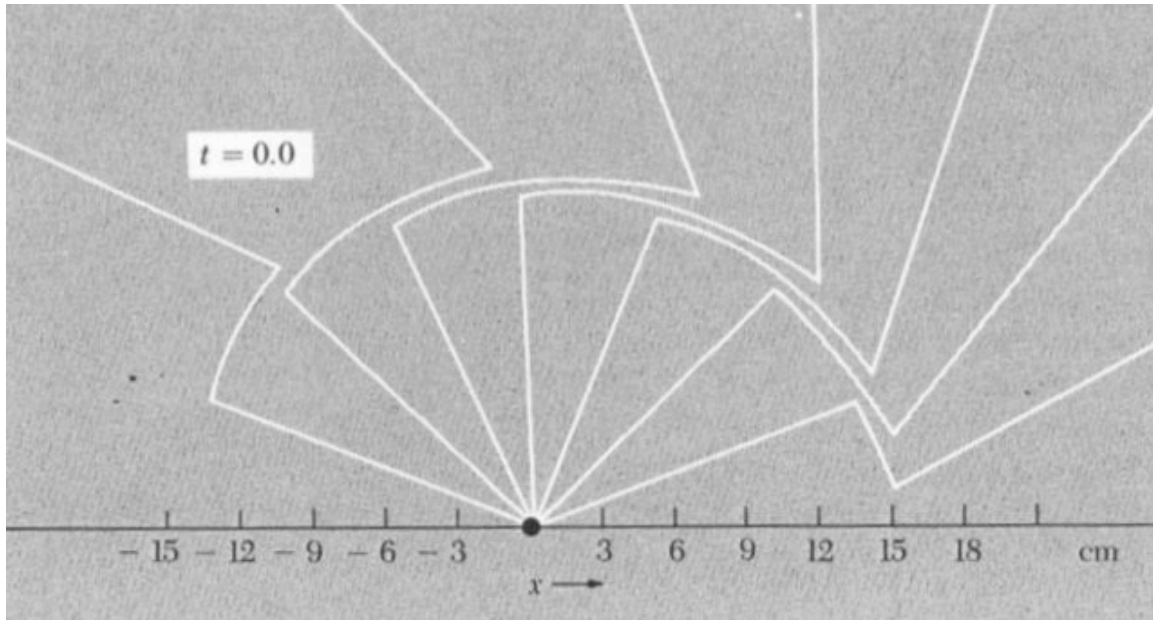


**Final Exam – Honors Physics**  
Distributed Friday April 30, 2004 Due Monday May 3, 2004  
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**SOLUTIONS**

**Problem 1 (25 points)**



The figure above shows the electric field lines due to an electron at time  $t = 0$ . Here is what I can tell you – the electron may have been at rest and then suddenly moved with constant velocity  $u$  along the  $x$ -axis or it was moving with constant velocity  $u$  and then suddenly stopped. So what did happen, at what time did the transition occur and what is  $u$ ?

Where was the electron at  $t = -7.5 \times 10^{-10}$  s and at that instant what was the magnitude (specify the numerical value) of the electric field at  $x = 0$ ?

**Solution:**

It is clear from the drawing that the electron had been moving with velocity  $u$  and then came to rest at the origin. We see that at  $t=0$  light has traveled distance  $L = 15$  cm – we know this because the circle along which the field lines break has a radius of 15 cm. So that means that the electron came to rest at  $t = -L/c = -5 \times 10^{-10}$  cm. The field lines on the other side of the circle extrapolate to 12 cm, were the electron would have been had it not stopped. So the ratio of the speed of the electron to that of light is  $\beta = 12/15 = 0.8$ .

Now  $t = -7.5 \times 10^{-10} \text{ s}$  corresponds to  $\tau = t = 2.5 \times 10^{-10} \text{ s}$   $t = 2.5 \times 10^{-10} \text{ s}$  before the electron stopped and during that time it traveled a distance  $\beta c \tau = 6 \text{ cm}$ . The electric field at the origin at that instant is given by:

$$E = \frac{1}{4\pi\epsilon_0} \frac{(1 - \beta^2)e}{r^2}$$

Substituting values in the above yields:  $E = 1.44 \times 10^{-7} \text{ V / m}$

### Problem 2 (25 points)

A solid sphere of charge  $Q$  and radius  $R$  has a spherically symmetric volume charge density  $\rho(r)$  that varies linearly with distance  $r$  from the center of the sphere and  $\rho(0) = 0$ . In terms of  $Q$  and  $R$  how much work was required to assemble this sphere of charge?

#### Solution:

Since the density varies linearly with  $r$   $\rho(r) = Ar$  where  $A$  is a constant that can be found by realizing that the integral over the sphere has to yield  $Q$ . That is,

$$Q = \int_0^R \rho(r) 4\pi r^2 dr = 4\pi A \int_0^R r^3 dr = \frac{4\pi AR^4}{4}$$

Now we calculate the work done in assembling the charge. First we calculate the amount of charge in some small sphere of radius  $r$ :

$$q = \int_0^r \rho(r) 4\pi r^2 dr = \frac{4\pi Ar^4}{4}$$

Now on top of that place a shell of charge of radius  $r$  and thickness  $dr$  – that shell has charge  $dq = 4\pi Ar^3 dr$  and the work done to bring this shell onto the sphere is just

$$dW = k \frac{q dq}{r}$$

Now substitute for  $q$  and  $dq$  and integrate:

$$W = k \int_0^R \frac{(4\pi A)^2 r^6}{4} dr = \frac{4}{7} k \frac{Q^2}{R}$$

where of course we used  $4\pi A = 4Q / R^4$  and as a reminder:  $k = 1 / 4\pi\epsilon_0$

### Problem 3 (25 points)

For a circular coil of wire with  $N$  turns and radius  $R$  the magnetic field along the axis of the coil at a distance  $z$  from the center of the coil is given by:

$$B(z) = \frac{\mu_0 N I R^2}{2(R^2 + z^2)^{3/2}}$$

An arrangement of two identical coaxial parallel coils separated by distance  $R$  carrying the same current in the same direction is referred to as a Helmholtz coil producing a very uniform magnetic field along the axis. Suppose you design such an arrangement to produce a 50 gauss magnetic field midway between the two coils with  $R = 1 \text{ m}$ . You have wire that will carry 5 A with a resistance per length of  $5 \Omega / \text{km}$ . You will use a single length of wire to wind both coils. How much power will be dissipated by these coils?

If the coils are separated by 2 m with the current kept at 5 A, what will be the value of the field midway between the coils?

#### Solution:

Using the above expression we can find an expression for the magnetic field between the coils a distance  $z$  from one of the coils when the coils are separated by distance  $s$ :

$$B(z,s) = \frac{\mu_0 N I R^2}{2} \left[ \frac{1}{(R^2 + z^2)^{3/2}} + \frac{1}{(R^2 + (s-z)^2)^{3/2}} \right]$$

To find the field at the center when  $s = R$  set  $z = R/2$  and you get:

$$B(z) = \frac{8\mu_0 N I}{5^{3/5} R} = 0.008992 \frac{N I}{R}$$

where  $B$  is in gauss. For  $R = 1 \text{ m}$  and  $B = 50 \text{ G}$   $N I = 5560 \text{ A-turns}$  or 1112 turns in each coil assuming a current of 5 A. The circumference of each coil is 6.28 m and so the total length of wire is about 14 km for a total resistance of 70 ohms. For a current of 5 A this works out to a power dissipation of 1.75 kW,

What happens to the central field when the spacing between the coils is increased to  $2R$ ? To find the change in the central field find the ratio:

$$\frac{B(R, 2R)}{B(R/2, R)} = 0.494$$

So the central field drops to 24.7 G.

**Problem 4 (25 points)**

Suppose that a particle of mass  $m$  and charge  $+q$  is subject to a uniform gravitational field such that  $\vec{F} = -mg\hat{k}$ . A horizontal uniform magnetic field is given by  $\vec{B} = B_0\hat{i}$ . Write down an expression for the velocity of the particle that will result in the particle moving in a straight line with speed  $u$ .

Now assume that the particle is held at rest at the origin of coordinates and released at time  $t = 0$ . Describe the motion of the particle and be as quantitative as possible. What is the maximum vertical excursion of the particle?

**Solution:**

The first part of the problem is simple – if the particle moves with constant velocity  $u$  along the  $-y$  direction where  $u$  is given by:

$$u = \frac{mg}{qB}$$

the net force is zero. For the case where the particle is released from the origin at  $t=0$  the problem is very similar to the case of the crossed E and B fields covered in a Note 006 only now instead of a constant force  $+qE$  along the y direction we have constant force  $-mg$  along the y direction. So this amounts to replacing  $E$  by  $-mg/B$  in all the equations. The particle follows the trajectory of a cycloid.