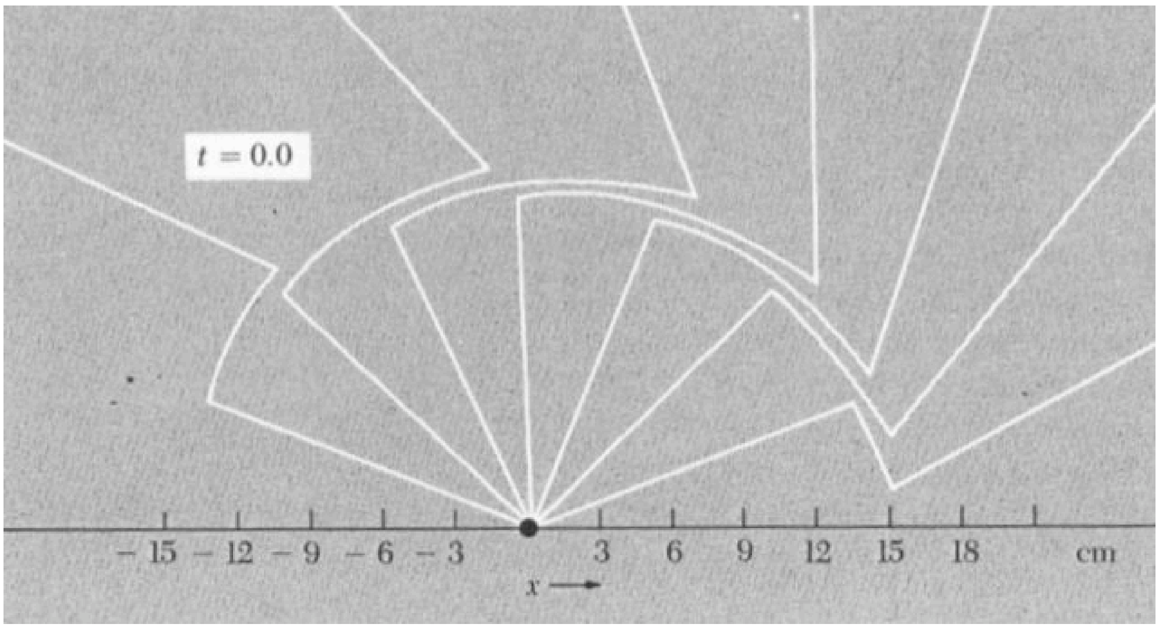
Final Exam – Honors Physics

Distributed Friday April 30, 2004   Due Monday May 3, 2004

Prof. Alex Dzierba

SOLUTIONS

Problem 1 (25 points)



The figure above shows the electric field lines due to an electron at time t = 0. Here is what I can tell you – the electron may have been at rest and then suddenly moved with constant velocity u along the x-axis or it was moving with constant velocity u and then suddenly stopped. So what did happen, at what time did the transition occur and what is u?

Where was the electron at $t = -7.5 \times 10^{-10}\ s$ and at that instant what was the magnitude (specify the numerical value) of the electric field at x = 0?

Solution:

It is clear from the drawing that the electron had been moving with velocity u and then came to rest at the origin. We see that at $t=0$ light has traveled distance L = 15 *cm* – we know this because the circle along which the field lines break has a radius of 15 *cm*. So that means that the electron came to rest at $t = -L / c = -5 \times 10^{-10}\ cm$. The field lines on the other side of the circle extrapolate to 12 *cm*, were the electron would have been had it not stopped. So the ratio of the speed of the electron to that of light is $\beta = 12 / 15 = 0.8$.

Now $t = -7.5\times10^{-10}\ s$ corresponds to $\tau = t = 2.5\times10^{-10}\ s\ t = 2.5\times10^{-10}\ s$ before the electron stopped and during that time it traveled a distance $\beta c\tau = 6\ cm$. The electric field at the origin at that instant is given by:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{\left(1-\beta^2\right)e}{r^2}$$

Substituting values in the above yields: $E = 1.44\times10^{-7}\ V/m$

Problem 2 (25 points)

A solid sphere of charge *Q* and radius *R* has a spherically symmetric volume charge density $\rho(r)$ that varies linearly with distance *r* from the center of the sphere and $\rho(0) = 0$. In terms of *Q* and *R* how much work was required to assemble this sphere of charge?

Solution:

Since the density varies linearly with *r* $\rho\left(r\right) = Ar$ where *A* is a constant that can be found by realizing that the integral over the sphere has to yield *Q*. That is,

$$Q = \int_0^R \rho\left(r\right)4\pi r^2 dr = 4\pi A\int_0^R r^3 dr = \frac{4\pi AR^4}{4}$$

Now we calculate the work done in assembling the charge. First we calculate the amount of charge in some small sphere of radius *r*:

$$q = \int_0^r \rho\left(r\right)4\pi r^2 dr = \frac{4\pi Ar^4}{4}$$

Now on top of that place a shell of charge of radius *r* and thickness *dr* – that shell has charge $dq = 4\pi Ar^3 dr$ and the work done to bring this shell onto the sphere is just

$$dW = k\frac{qdq}{r}$$

Now substitute for *q* and *dq* and integrate:

$$W = k\int_0^R \frac{\left(4\pi A\right)^2 r^6}{4}dr = \frac{4}{7}k\frac{Q^2}{R}$$

where of course we used $4\pi A = 4Q / R^4$ and as a reminder: $k = 1/4\pi\varepsilon_o$

Problem 3 (25 points)

For a circular coil of wire with N turns and radius R the magnetic field along the axis of the coil at a distance z from the center of the coil is given by:

$$B(z) = \frac{\mu_o NIR^2}{2\left(R^2 + z^2\right)^{3/2}}$$

An arrangement of two identical coaxial parallel coils separated by distance R carrying the same current in the same direction is referred to as a Helmholtz coil producing a very uniform magnetic field along the axis. Suppose you design such an arrangement to produce a 50 *gauss* magnetic field midway between the two coils with R = 1 *m*. You have wire that will carry 5 *A* with a resistance per length of $5\ \Omega / km$. You will use a single length of wire to wind both coils. How much power will be dissipated by these coils?

If the coils are separated by 2 *m* with the current kept at 5 *A*, what will be the value of the field midway between the coils?

Solution:

Using the above expression we can find an expression for the magnetic field between the coils a distance z from one of the coils when the coils are separated by distance s:

$$B(z,s) = \frac{\mu_o NIR^2}{2}\left[\frac{1}{\left(R^2 + z^2\right)^{3/2}} + \frac{1}{\left(R^2 + (s-z)^2\right)^{3/2}}\right]$$

To find the field at the center when $s = R$ set $z = R/2$ and you get:

$$B(z) = \frac{8\mu_o NI}{5^{3/5} R} = 0.008992 \frac{NI}{R}$$

where B is in *gauss*. For R = 1 *m* and B = 50 *G* NI = 5560 *A-turns* or 1112 turns in each coil assuming a current of 5 *A*. The circumference of each coil is 6.28 *m* and so the total length of wire is about 14 *km* for a total resistance of 70 ohms. For a current of 5 *A* this works out to a power dissipation of 1.75 *kW*,

What happens to the central field when the spacing between the coils is increased to 2*R* ? To find the change in the central field find the ratio:

$$\frac{B(R,2R)}{B(R/2,R)} = 0.494$$

So the central field drops to 24.7 *G*.

Problem 4 (25 points)

Suppose that a particle of mass *m* and charge +*q* is subject to a uniform gravitational field such that $\vec{F} = -mg\hat{k}$. A horizontal uniform magnetic field is given by $\vec{B} = B_o\hat{i}$. Write down an expression for the velocity of the particle that will result in the particle moving in a straight line with speed *u*.

Now assume that the particle is held at rest at the origin of coordinates and released at time *t* = 0. Describe the motion of the particle and be as quantitative as possible. What is the maximum vertical excursion of the particle?

Solution:

The first part of the problem is simple – if the particle moves with constant velocity *u* along the –y direction where *u* is given by:

$$u = \frac{mg}{qB}$$

the net force is zero. For the case where the particle is released from the origin at *t=0* the problem is very similar to the case of the crossed E and B fields covered in a Note 006 only now instead of a constant force +*qE* along the y direction we have constant force –*mg* along the y direction. So this amounts to replacing *E* by –*mg*/*B* in all the equations. The particle follows the trajectory of a cycloid.