Quiz 2

Honors Physics

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Open notes and book - take 30 minutes - print your name in upper left corner

Amprere's law states that:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int \vec{J} \cdot \hat{n} da \tag{1}$$

where the integral on the left hand side of the equation is taken over a closed path and the integral on the right hand side is taken over a surface bounded by the closed path. The direction of the normal ($\hat{n}$) to the surface and the direction in which the path integral is taken are related by the right hand rule. The vector $\vec{J}$ is the current density – current per area.

As discussed in class, equation 1 needs to be modified in order to take into account conservation of charge. When modified, the correct equation is:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int \vec{J} \cdot \hat{n} da + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot \hat{n} da \tag{2}$$

Now consider the following situation: a sphere of radius R whose total charge at any given time is $Q(t)$. The charge is positive and is always uniformly distributed on the surface of the sphere and is continually leaking away so that the leaking charge forms a current that is along the radius of the sphere symmetric over the surface of the sphere. Think of this, if you will, as a radioactive sphere. The magnitude of dQ/dt is I_0.

(a) On the surface of the sphere, what is $|\vec{J}|$?

SOLUTION

$$|\vec{J}| = \frac{I_0}{4\pi R^2} \tag{3}$$

(b) Imagine drawing a very small circular loop of area Δa on the surface of the sphere. For this loop, what would be the value of the right hand side of equation 1?

SOLUTION

$$\mu_0 \int_{loop} \vec{J} \cdot \hat{n} da = \mu_0 \frac{I_0}{4\pi R^2} \Delta a \tag{4}$$

and note that this positive assuming the normal to the surface of the sphere points away from the center of the sphere.

(c) For this same imaginary loop what can you say about the left hand side of equation 1? Explain your answer. *Hint: think about the symmetry of the problem.*

SOLUTION

It has to be zero. If there is a $\vec{B}$ field it would have to lie on the surface of the sphere but by symmetry there cannot be any $\vec{B}$ field.

(d) The reasoning you give for part (c) should also apply to left hand side of equation 2. Again referring to our small loop, what is the right hand side of equation 2? Carefully explain your answer taking into account the second term on the right hand side.

SOLUTION

For the left hand side of equation 2 we already evaluated the first term (see the answer to part (b)). For the second term:

$$\mu_0 \epsilon_0 \frac{\partial}{\partial t} \int_{loop} \vec{E} \cdot \hat{n} da = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \Delta a \tag{5}$$

The minus sign reflects that the electric field is decreasing since the charge is decreasing. Recalling that $E = Q/4\pi\epsilon_0 R$ and $I_0 = dQ/dt$:

$$-\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \Delta a = -\mu_0 \frac{I_0}{4\pi R^2} \Delta a \tag{6}$$

So for our little loop:

$$\mu_0 \int \vec{J} \cdot \hat{n} da + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot \hat{n} da = \mu_0 \frac{I_0}{4\pi R^2} \Delta a - \mu_0 \frac{I_0}{4\pi R^2} \Delta a = 0 \tag{7}$$

(e) Briefly, what is your overall conclusion?

SOLUTION

You really need that second term in Ampere's law. This was a cool quiz.