Honors Physics – P222

Exam III

Distributed on Friday, April 11, 2003
and due at 1:25 pm on Monday, April 14, 2003

SOLUTIONS

Thank you,

Alex

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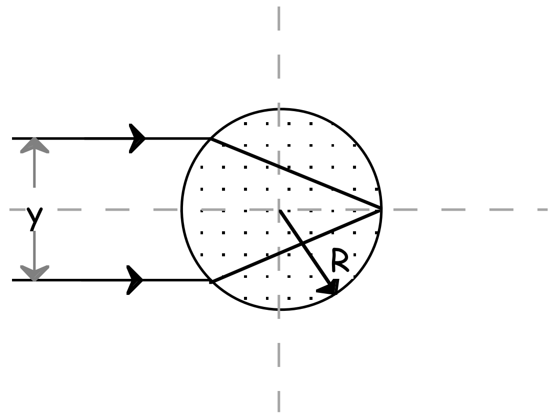
Problem 1 (20 points)

Two parallel beams of light are separated by distance y and fall normally and symmetrically on a rod of glass with circular cross-section of radius R and meet at a focus at the opposite circumference of the rod as shown in the drawing. The index of refraction of the glass is 1.6. Find the ratio y/R (give a number).

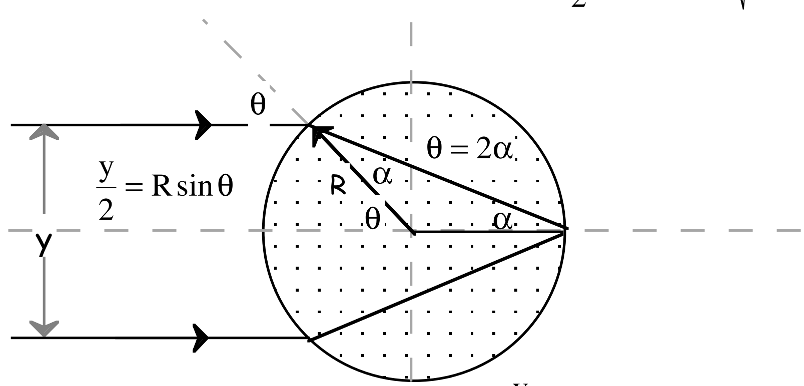


Solution:

$$\sin\theta = n\sin\alpha$$

$$2\sin\alpha\cos\alpha = n\sin\alpha$$

$$\Rightarrow \cos\alpha = \frac{n}{2} \Rightarrow \sin\alpha = \sqrt{1-\frac{n^2}{4}}$$

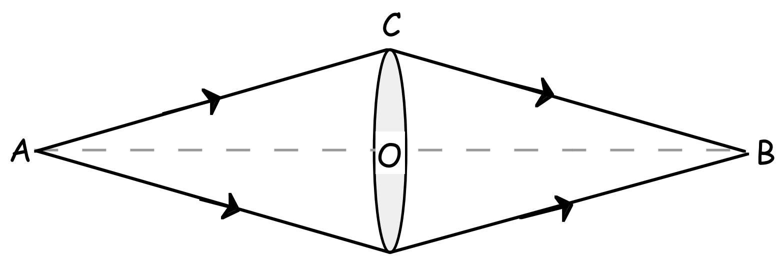


$$\frac{y}{R} = 2\sin\theta = 2n\sin\alpha$$

$$= 2n\sqrt{1-\frac{n^2}{4}} = 1.92$$

Problem 2 (20 points)

The figure below (not to scale) shows light from point A focused at point B with a converging lens of index of refraction n = 1.6. The lens is symmetrically placed midway between points A and B. The distance between A and B is 2 m and the height of the lens is 20 cm. The thickness of the lens at the edges is 3 mm. What is the thickness of the lens at its center?



Solution:

The time for light traveling paths ACB and AOB should be the same.

For ACB: $t = \dfrac{2\sqrt{1+0.1^2} - 0.003}{c} + \dfrac{0.003n}{c}$

For AOB: $t = \dfrac{2-x}{c} + \dfrac{xn}{c}$ where x is the thickness of the lens at the center.

Setting these two equal:

$$2\sqrt{1+0.1^2} - 2 + 0.003(n-1) = x(n-1)$$
$$n - 1 = 0.6$$
$$\Rightarrow x = 0.02\text{m}$$

Problem 3 (20 points)

A long straight copper wire of cross-sectional radius equal to 2 mm is carrying 10 A of current. What is the numerical value of the flux of the Poynting vector integrated over the surface of the wire for each meter of wire? The resistivity of copper is: $\rho = 1.7 \times 10^{-8} \;\; \Omega \cdot m$.

Solution:

The flux of the Poynting vector over 1 m length of wire is just I^2R where R is the resistance of 1 m of wire and that is given by:

$$R = \rho \frac{L}{A} = 1.7 \times 10^{-8} \Omega \cdot m \frac{1.0m}{\pi \left(2 \times 10^{-3}\right)^2 m^2}$$

So:

$$I^2R = \frac{1.7}{4\pi} W = 0.135W$$

Problem 4 (20 points)

A converging lens of focal length 4 cm is followed by a diverging lens with focal length 8 cm and the separation between the two is 6 cm. The diverging lens is then followed by a converging lens with focal length *f*. The separation between the diverging lens and second converging lens is 1.4 cm. We want parallel rays of light incident on this system to leave the system still parallel. What should the focal length of the second converging lens be? *Hint: use the matrix technique*.

Solution:

Multiply these matrices:

$$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1.4 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1/8 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -1/4 & 1 \end{pmatrix}$$

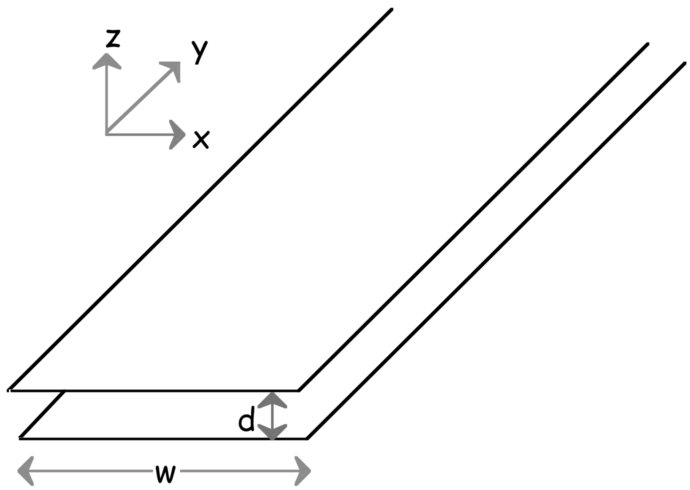
Multiply by hand or use *Mathematica*: The answer: $\begin{pmatrix} -0.93751 & 8.45 \\ -0.3125 + \frac{0.9375}{f} & \frac{7}{4} - \frac{8.45}{f} \end{pmatrix}$

This is in the form $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$. Parallel rays to parallel rays means C=0 since this will give:

$\theta_2 = D \cdot \theta_1$ and in our face that implies that f=3 cm

Problem 5 (20 points)

One end of a parallel plate transmission line is shown. This transmission line extends along the y axis and a voltage pulse applied from one plate to the other at this end of the line propagates down the transmission line along the y direction. You can make the reasonable and simplifying assumption that the **E** and **B** fields due to the voltage and current exist only in the space between the plates and that the current in a given plate is uniform along the width *w*.



(a) How do the directions of **E**, **B** and the Poynting vector, **P**, depend on the polarity of the voltage from the bottom to the top plate?
(b) What is the capacitance per length (along the y-direction) of this line?
(c) What is the inductance per length (along the y-direction) of this line?
(d) Assume that at the far end of this line we connect the top and bottom plates with a slab of copper so that the resistance due to the slab of copper equals the impedance of the line. One dimension of the slab is *w*, another is *d*, the separation between the plates and the other is *t*, the thickness of the slab. Find a numerical value for *t* assuming that the resistivity of copper is $\rho = 1.7 \times 10^{-8} \ \ \Omega \cdot m$. Comment on the practicality of such a slab.

Solution: on next page

For part [a]: The **E** field points from high potential to low potential and depends on polarity as does the **B** field which depends on the direction of current but $\mathbf{E} \times \mathbf{B}$ points along the transmission line (y-axis) away from the power source.

For part [b]: Imagine a length of the transmission line along the y-axis. Call the length ℓ. The capacitance of that length is the standard capacitance for a parallel plate capacitor with area A and separation d:

$$C = \frac{\varepsilon_o A}{d} = \frac{\varepsilon_o \ell w}{d} \Rightarrow C_o = \frac{C}{\ell} = \varepsilon_o \frac{w}{d}$$

For part [c]: The **B** field is parallel to the plates and oriented along the x-axis. From Ampere's law:

$$B = \mu_o \frac{I}{w}$$

For a given length ℓ of the transmission line along the y-axis we have volume $\ell \cdot d \cdot w$ between the plates so the energy stored in the magnetic field is just:

$U = \frac{B^2}{2\mu_o} \ell dw = \frac{1}{2} \mu_o \ell \frac{d}{w}$ and setting $U = \frac{1}{2} LI^2$ we have $L_o = \frac{L}{\ell} = \mu_o \frac{d}{w}$

Notice that $\frac{1}{\sqrt{L_o C_o}} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = c$

For part [d]: We need to find the impedance which is: $Z_o = \sqrt{\frac{L_o}{C_o}} = \frac{d}{w}\sqrt{\frac{\mu_o}{\varepsilon_o}} = \frac{d}{w} 377\Omega$

The resistance of a slab of copper of length d and cross-section area w times t where t is the thickness is: $R = \rho \frac{d}{wt}$ and setting $R = Z_o \Rightarrow t = \frac{\rho}{377\Omega} \approx 0.5 \times 10^{-10}\,\text{m}$ or about the dimensions of an atom!