Honors Physics – P222

Solutions to
Exam I

Distributed on Friday, February 7, 2003
and due at 1:25 pm on Monday, February 10, 2003

Guidelines:

(1) This exam consists of five problems, each worth 20 points. Please show all your work in the blue books provided.

(2) You can use your text, notes and computer – but you may not consult with anyone while taking this exam.

(3) If you have a question about any of these problems please send me an e-mail. I will try to respond as quickly as possible.

(4) Please take the time to write your solutions neatly and clearly. Be sure to specify units.

Thank you,

Alex

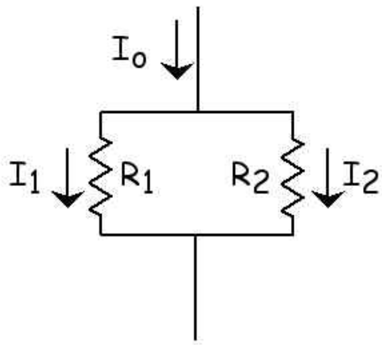
Alex R. Dzierba

Please print your name below and also sign your name. By so doing you are stating your understanding of the rules under which this exam is given and that you followed these rules.

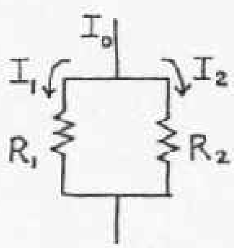
____

Problem 1 (20 points)

In the figure shown, current I_o splits into I_1 and I_2 with $I_o = I_1 + I_2$. Find the relationship between I_o and I_1 and the resistances starting with the requirement that the total power dissipation in the two resistors is a minimum.



Solution:

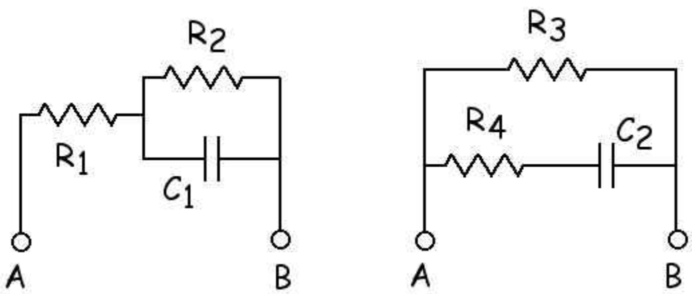


$$P = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_1 + (I_o - I_1)^2 R_2$$
$$= I_1^2 (R_1 + R_2) - 2 I_o R_2 I_1 + R_2 I_o^2$$
$$\frac{dP}{dI_1} = 2I_1 (R_1 + R_2) - 2 I_o R_2$$

To minimize P, set $\frac{dP}{dI_1} = 0$ This gives the condition $I_1 = I_o \frac{R_2}{R_1 + R_2}$, which we also get from Ohm's law. Observe that P is minimum, not maximum, for $dP/dI_1 < 0$ if I_1 is less than $I_o R_2/(R_1 + R_2)$

Problem 2 (20 points)

Consider the two circuits below. Suppose we want the impedance between terminals A and B for the two circuits to be equal in the two extremes: $\omega \to 0$ and $\omega \to \infty$. Find R_3 and R_4 each in terms of R_1 and R_2.



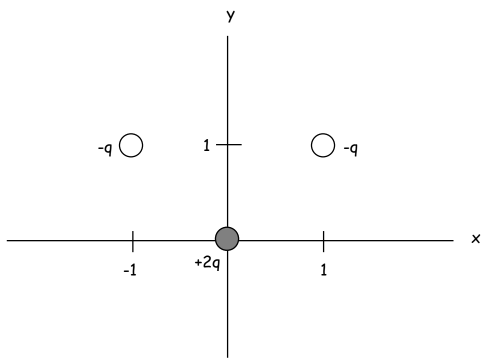
Solution:

Recall that in the very low frequency limit replace the capacitors by an open circuit and in the limit of very high frequency replace the capacitors by wires (short ciruit).

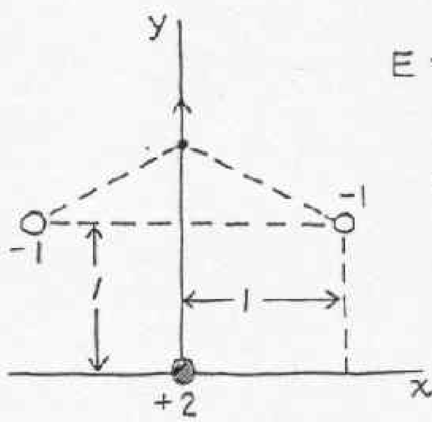
To make these circuits equivalent when $\omega \to 0$, $\boxed{R_3 = R_1 + R_2}$
Equivalence for $\omega \to \infty$ requires $R_1 = R_3 R_4/(R_3 + R_4)$.
Solving for R_4: $\boxed{R_4 = R_1(R_1 + R_2)/R_2}$ If circuits are

Problem 3 (20 points)

The figure shows a positive charge of +2q at the origin of coordinates and charges –q at (x,y) = (+1,+1) and (-1,+1). Find the point along the y-axis where the electric field vanishes. Find a numerical answer accurate to at least 4 figures. *Hint: Think numerical iteration.*



Solution:



$$E = \frac{2}{y^2} - \frac{2}{(y-1)^2+1} \times \frac{y-1}{\sqrt{(y-1)^2+1}}$$

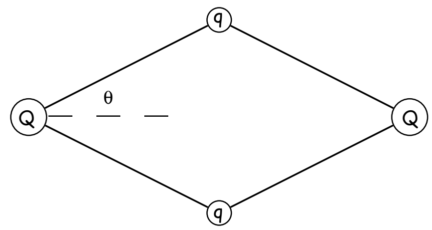
$$= 0 \text{ if } \frac{1}{y^4} = \frac{(y-1)^2}{(y^2-2y+2)^3}$$

$$y^4 = \frac{(y^2-2y+2)^3}{(y-1)^2}$$

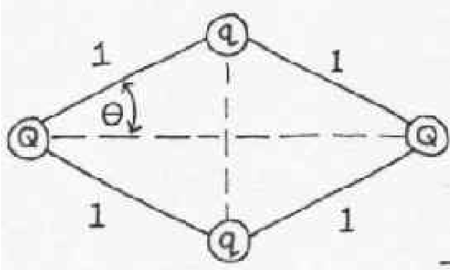
Solve by iteration: We know y must be > 1, so let's start with y = 2 on the right. After 6 iterations we converge to y = 1.6207 ... with no further change in the 5th figure.

Problem 4 (20 points)

The figure shows 4 charges, two with charge +q and two with charge +Q. They are connected by four unstretchable strings and the entire system is in equilibrium. No external forces are present. Find an expression for θ in terms of q and Q.



Solution:



Tension must be same in all strings. Call it T. q is in equilibrium if $q^2/(2\sin\theta)^2 = 2T\sin\theta$. Similarly, force on Q is zero if $Q^2/(2\cos\theta)^2 = 2T\cos\theta$. Together these equations give $Q^2\sin^3\theta = q^2\cos^3\theta$ or $q^2/Q^2 = \tan^3\theta$.

Problem 5 (20 points)

An infinitely long cylinder is filled with charge where the charge per volume $\rho(r)$ varies with distance r from the axis of the cylinder like $\rho(r) = C \cdot r$ (C is a constant). The radius of the cylinder is A and the charge per length is λ.

a. Find the constant C in terms of λ and A.
b. Find the E field as a function of r in the region $r > A$
c. Find the E field as a function of r in the region $0 < r < A$

Solution:

Part (a): In a length L of the cylinder the enclosed charge is λL. We can also find the charge by integrating the charge density $\rho(r)$. In doing this integral from $r = 0$ to $r = a$ the volume element is $dV = 2\pi rL \cdot dr$. We have two ways of finding charge and therefore: $Q = \lambda L = \int_0^A \rho(r) 2\pi rL \cdot dr = 2\pi CL \int_0^A r^2 dr \;\Rightarrow\; C = \dfrac{3\lambda}{2\pi A^3}$

Part (b): Outside the cylinder the field looks like that of a thin line of charge along the axis of the cylinder so: $E = \dfrac{\lambda}{2\pi\varepsilon_o r}$

Part (c): Now imagine as a Gaussian surface a cylinder of length L and radius r where $r < A$. The flux of E over the Gaussian surface is $\Phi = E \cdot 2\pi rL = Q / \varepsilon_o$ where Q is the charge enclosed by the surface which is just the expression given for part (a) above but where the upper limit of the integral is now r instead of A. Substitution yields:

$$E = \frac{Cr^2}{3\varepsilon_o} = \frac{\lambda}{2\pi\varepsilon_o} \frac{r^2}{A^3}$$