Exam 1 Solutions

Honors Physics

Alex R. Dzierba

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RULES:

- This is an open-book, open-notes take-home exam.
- Show all your work *clearly* in the blue books.
- The exam is due Monday, February 16 at 1:25 pm.
- You may not consult with anyone about this exam starting noon on February 12 until you turn it in on February 16.

Problem 1

Final numerical answers are expected here – be sure to specify units.

Figure 1(a) shows a capacitor $C = 0.01\ \mu F$ and inductor $L = 700\ mH$ connected by a switch S that will be closed at $t = 0$. Before the switch is closed the capacitor carries charge of 6 μC.

(a) How soon, after the switch is closed, is the charge on the capacitor equal to zero? (b) What is the maximum current in the inductor?

Figure 1(b) is a modification of Figure 1(a) – a resistor, $R = 1200\ \Omega$, is added. Again, before the switch is closed the capacitor carries charge of 6 μC and the switch is closed at $t = 0$.

(c) What is the difference in the oscillation frequency of this circuit and the oscillation frequency of the circuit of Figure 1(a)? (d) If R is a variable resistor, for what range of R (if any) will there be *no* oscillations after the switch is closed – assuming L and C are fixed at their current values?

Problem 1 - Solution

When the switch is closed the circuit will oscillate at a frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{1}$$

The charge will first go to zero at one-fourth of the period corresponding to this frequency or $\pi\sqrt{LC}/2$ or 0.13 ms.

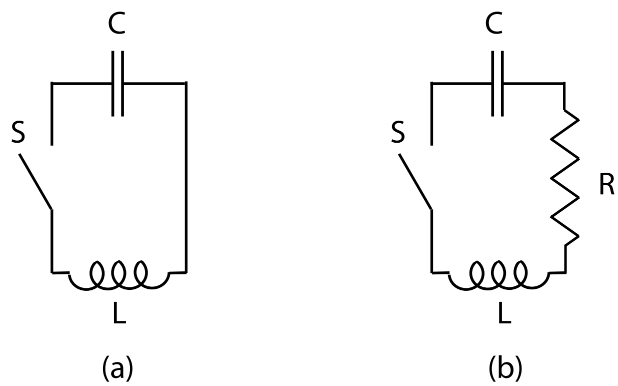

Figure 1: Problem 1

Energy is conserved and the total energy is the energy in the capacitor given by $q^2/2C$ and the energy in the inductor given by $Li^2/2$. When the capacitor has maximum charge the current in the inductor is zero and vice-versa. The initial charge on the capacitor is the maximum charge. So the maximum current is given by requiring:

$$\frac{1}{2}Li_{max}^2 = \frac{q_{max}^2}{2C} \tag{2}$$

or

$$i_{max} = \frac{q_{max}}{\sqrt{LC}} \tag{3}$$

or 0.07 A. To answer part (c) recall Equation 13 of Note 5:

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}} \tag{4}$$

and of course $f = \omega/2\pi$. For the case without the resistor f_0 is 1902 Hz and including the resistor the frequency is 1897 Hz.

From equation 4 we can also see that if $\omega_0^2 < R^2/4L^2$ there will be no oscillations. So if L and C are fixed this means:

$$R > 2\sqrt{\frac{L}{C}} \tag{5}$$

or that R is greater than 16.7 KΩ.

Problem 2

Consider a spherical non-conducting shell of inner radius a and outer radius b. Between $r = a$ and $r = b$ the density of charge is given by $\rho(r) = A \cdot r$ where r is measured from the center of the shell and A is a constant. On the inside surface of the shell is a thin uniform layer of charge with $-\sigma_o$ charge per area and on the outside surface there is another this layer of charge with $+\sigma_o$ charge per area. The total charge of the sphere is Q.

(a) In terms of Q and σ_o what is A?

What is the electric field:

(b) Just outside $r = b$ and just inside $r = b$? (c) Just outside $r = a$ and just inside $r = a$?

Problem 2 - Solution

The total charge on the sphere is that due to the two layers of charge plus the integral of the charge density. Let's call Q_1 the charge on the inside surface, Q_3 the charge on the outside surface and Q_2 the charge in between. Then we have:

$$Q_1 = -4\pi\sigma_o a^2 \tag{6}$$

$$Q_3 = +4\pi\sigma_o b^2 \tag{7}$$

$$Q_2 = \int_a^b \rho(r) 4\pi r^2 dr = A4\pi \int_a^b r^3 dr = \pi A \left(b^4 - a^4\right) \tag{8}$$

These add to give Q so:

$$A = \frac{Q + 4\pi\sigma_o(a^2 - b^2)}{\pi(b^4 - a^4)} \tag{9}$$

The field just outside $r = b$ is that of a point charge Q a distance b away kQ/b^2. There is a discontinuity of σ_o/ϵ_o as we go just inside $r = b$ so there the field is $kQ/b^2 - \sigma_o/\epsilon_o$.

Just inside $r = a$ the electric field is zero. Again we have a discontinuity and just outside $r = a$ the field is σ_o/ϵ_o – and of course directed along the radius and pointing in towards the center.

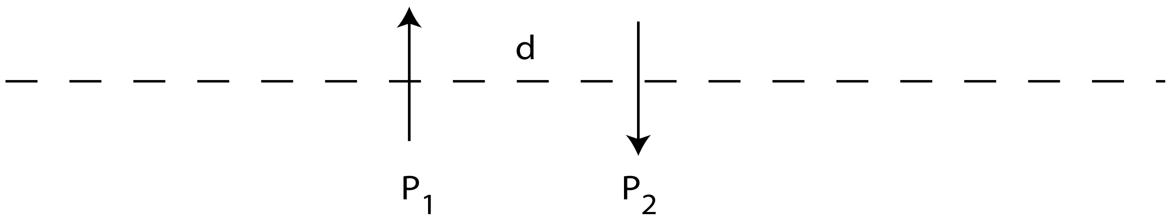

Figure 2: Problem 3

Problem 3

See Figure 2. Two electric dipoles, $\vec{p}_1$ and $\vec{p}_2$ are anti-parallel and separated by a distance d. At what point(s) along a line perpendicular to the dipoles and passing through their centers is the total electric field zero?

Problem 3 - Solution

For simplicity place $\vec{p}_1$ at the $x = 0$ and $\vec{p}_2$ at $x = d$. Along the x-axis the fields from the two dipoles are anti-parallel. Recall that along a perpendicular axis to a dipole p the magnitude of the electric field is kp/r^3 at distance r from the dipole. Assuming that the field vanishes at $x = -s$ the condition that the field vanish involves solving:

$$\frac{p_1}{s^3} - \frac{p_2}{(s+d)^3} = 0 \tag{10}$$

The solution for which is:

$$s = \frac{dp_1^{1/3}}{p_2^{1/3} - p_1^{1/3}} \tag{11}$$

and only is a solution if $p_2 > p_1$. Assuming that the field vanishes for $x = s > d$ we require:

$$\frac{p_1}{s^3} - \frac{p_2}{(s-d)^3} = 0 \tag{12}$$

The solution for which is:

$$s = \frac{dp_1^{1/3}}{p_1^{1/3} - p_2^{1/3}} \tag{13}$$

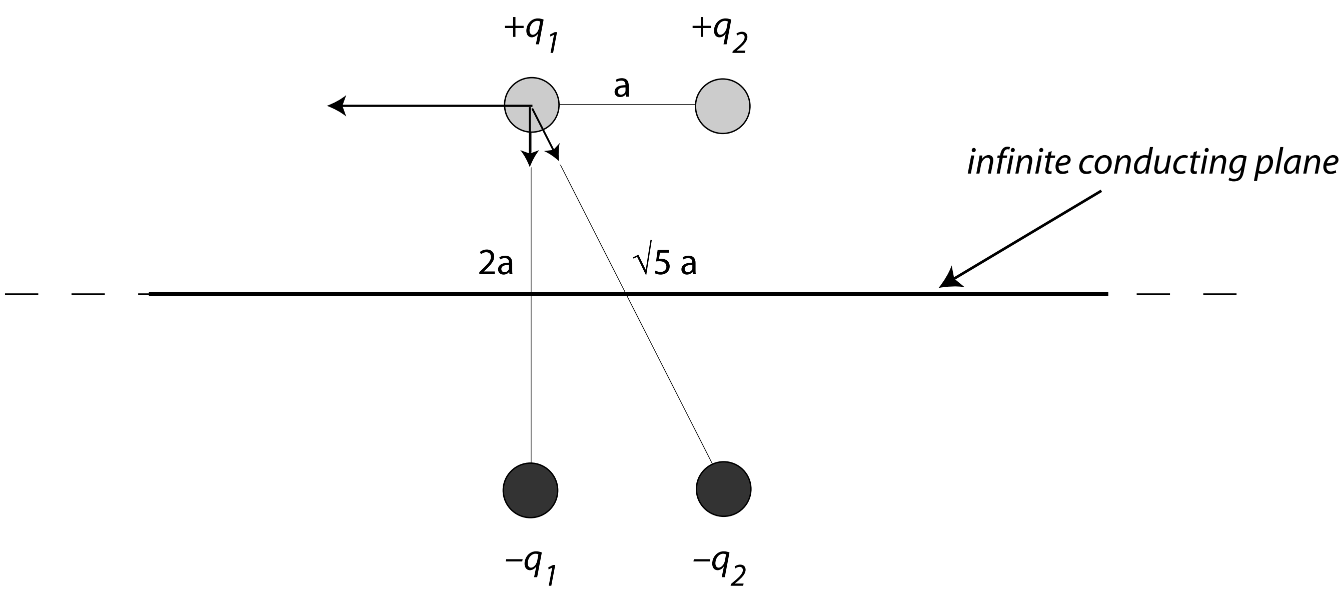

Figure 3: Problem 4

and only is a solution if $p_1 > p_2$. In between the two dipole the vanishing field requirement is:

$$\frac{p_1}{s^3} - \frac{p_2}{(d-s)^3} = 0 \tag{14}$$

The solution for which is:

$$s = \frac{dp_1^{1/3}}{p_1^{1/3} + p_2^{1/3}} \tag{15}$$

To review: along the x-axis the net electric field vanishes at $x = \pm\infty$. If the dipoles are equal then there is only one other place where the field vanishes – halfway between the dipoles. If the dipoles are not equal the field vanishes in between the dipoles as given by equation 15 and to the left of $\vec{p}_1$ or to the right of $\vec{p}_2$ depending on the relative magnitudes of the dipoles.

Problem 4

Two charges, $+q_1$ and $+q_2$ are each held a distance a above an infinite conducting plane. The charges are also separated by distance a.

(a) Ignoring the forces holding the charges in place what is the total electrical force on each charge?
(b) What force, if any, do the two charges exert on the infinite conducting plane?

Problem 4 - Solution

Figure 3 shows the position of the image charges. We show the forces on one of the charges (q_1). Let's look at the total force on this charge in unit vector notation:

$$\vec{F}_1 = -\frac{kq_1q_2}{a^2}\left(1-\frac{1}{5^{2/3}}\right)\hat{\mathbf{i}} - \frac{kq_1}{a^2}\left(\frac{q_1}{4}+\frac{2q_2}{5^{3/2}}\right)\hat{\mathbf{j}} \tag{16}$$

And the force on charge q_2 is:

$$\vec{F}_2 = \frac{kq_1q_2}{a^2}\left(1-\frac{1}{5^{3/2}}\right)\hat{\mathbf{i}} - \frac{kq_2}{a^2}\left(\frac{q_2}{4}+\frac{2q_1}{5^{3/2}}\right)\hat{\mathbf{j}} \tag{17}$$

The force on the conducting plane:

$$\vec{F}_{plane} = \frac{k}{a^2}\left(\frac{q_1^2+q_2^2}{4}+\frac{4q_1q_2}{5^{3/2}}\right)\hat{\mathbf{j}} \tag{18}$$