

Special Problems
Honors Physics – Week 4
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Problem 4.1

See Figure 1(a). An infinite planar sheet of uniform charge per area σ lies in the $y - z$ plane at $x = 0$. Next to the sheet is a slab of uniform volume charge density ρ that is parallel to the $y - z$ plane and extends from $x = 0$ to $x = d$. Find the electric field as a function of x in these three regions: (a) $x < 0$; (b) $0 < x < d$; and (c) $x > d$. Plot the value of the electric field as a function of x using the convention that the field is positive if \vec{E} points along the positive x axis and negative if \vec{E} points along the negative x axis. Comment on discontinuities (if any) at $x = 0$ and $x = d$.

Problem 4.2

See Figure 1(b). A sphere of radius R is filled with charge of uniform volume charge density ρ . A spherical cavity is carved out of this charge. The cavity has radius $R/2$ and the edge of the cavity passes through the center of the large sphere.

Find the electric field at points A , B and C where A is the center of the large sphere and B is the center of the cavity. Now here's a real challenge – calculate the field everywhere inside the cavity.

Problem 4.3

In this problem you will be introduced to the concept of the vector operator $\vec{\nabla}$ also referred to the *del* operator. Here is the definition:

$$\vec{\nabla} = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z} \quad (1)$$

Suppose $f(x, y, z)$ is a *scalar* field and $\vec{A}(x, y, z)$ is a vector field. You should appreciate that each component of \vec{A} is a function of the coordinates:

$$\vec{A}(x, y, z) = A_x(x, y, z)\hat{\mathbf{i}} + A_y(x, y, z)\hat{\mathbf{j}} + A_z(x, y, z)\hat{\mathbf{k}} \quad (2)$$

When $\vec{\nabla}$ operates on a scalar function, a vector field emerges called the *gradient* of f :

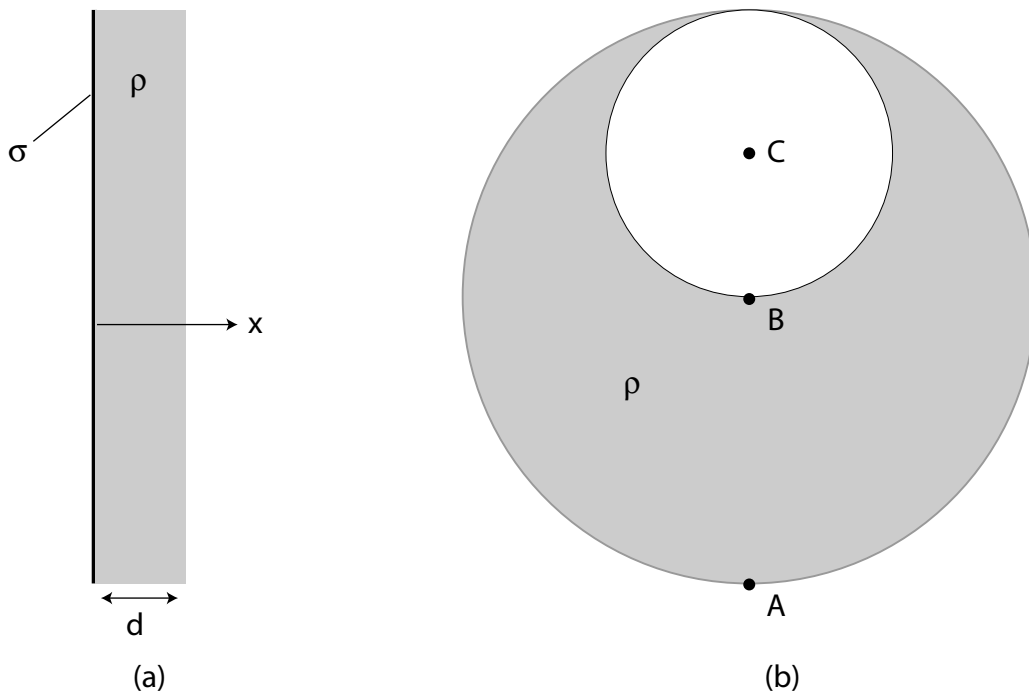


Figure 1: (a) Problem 4.1; (b) Problem 4.2

$$\vec{\nabla} f = \hat{\mathbf{i}} \frac{\partial f}{\partial x} + \hat{\mathbf{j}} \frac{\partial f}{\partial y} + \hat{\mathbf{k}} \frac{\partial f}{\partial z} \quad (3)$$

When $\vec{\nabla}$ operates on a vector there are two possibilities, similar to the product of two vectors: the *dot* product and *cross* product. The *divergence* of \vec{A} is like a dot product and it yields a scalar:

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (4)$$

The *curl* of \vec{A} is like a cross product and it yields a vector. Here's the determinant form of the cross product, written as $\vec{\nabla} \times \vec{A}$

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

OK - now for the problem. Suppose:

$$f(x, y, z) = 3x^3y^2z - 2xy - 6yz \quad (5)$$

and

$$\vec{A} = (x^2 - y^2)\hat{\mathbf{i}} + (3x^2z)\hat{\mathbf{j}} + (2xy)\hat{\mathbf{k}} \quad (6)$$

Now find the following:

1. $\vec{\nabla} f$
2. $\vec{\nabla} \cdot \vec{A}$
3. $\vec{\nabla} \times \vec{A}$
4. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A})$
5. $\vec{\nabla}(\vec{\nabla} \cdot \vec{A})$
6. $\vec{\nabla} \times \vec{\nabla} f$

After you compute these take a look at your answers to parts 4 and 6 above. Would you get the same answer for any arbitrary f and \vec{A} ?