Special Problems for Week 10
Honors P222
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March 24-28, 2003
Fixed: March 25, 2003

Problem 1

Using the definitions for the gradient, divergence and curl operators in Cartesian coordinates prove the following for any vector field **A** and scalar function *f*:

$$\vec{\nabla} \times \left(\vec{\nabla} f \right) = 0$$

$$\vec{\nabla} \cdot \left(\vec{\nabla} \times \mathbf{A} \right) = 0$$

$$\vec{\nabla} \times \vec{\nabla} \times \mathbf{A} = \vec{\nabla}\left(\vec{\nabla} \cdot \mathbf{A} \right) - \vec{\nabla}^2 \mathbf{A}$$

Problem 2

In a region of space where there are no charges $\left(\rho(x, y, z) = 0 \right)$ or currents $\left(\mathbf{J}(x, y, z) = 0 \right)$ Maxwell's equations are:

$$\vec{\nabla} \cdot \mathbf{E} = 0$$

$$\vec{\nabla} \cdot \mathbf{B} = 0$$

$$\vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\vec{\nabla} \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

(a) Using the last identity listed in Problem 1 and the above Maxwell's equations, show that these wave equations follow:

$$\vec{\nabla}^2 \mathbf{B} = \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$\vec{\nabla}^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

In the notes from the Feynman Lectures and from class, we showed that for a plane wave propagating along the +*x* direction, **E** and **B** are in the y-z plane and perpendicular to each other such that $\mathbf{E}\times\mathbf{B}$ is along the +*x* direction. If we align the *y* axis along **E** then the above wave equations become:

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$$
$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

Problem 3

Show that

$$\mathrm{B_z(x,t) = B_o \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{f}t\right)}$$
$$\mathrm{E_y(x,t) = E_o \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)}$$

satisfy the above wave equation (as listed in Problem 3) assuming a certain relationship between λ, *T* and *c* (wavelength, period and speed of light). What is that relationship?

Problem 4

A coaxial cable consists of a conducting cylinder of radius *b* and a wire of radius *a* along the axis of the cylinder. Assuming that the space between the two conductors can be treated as a vacuum, find the capacitance per length (C_o) and inductance per length (L_o) of this arrangement and verify that:

$$c = \frac{1}{\sqrt{C_o L_o}}$$

Problem 5

Using the results of Problem 4 and using the expression for the impedance of the cable:

$$Z_o = \sqrt{\frac{L_o}{C_o}}$$

find the outer radius of a coaxial cable for a $75\ \Omega$ cable with an inner conductor of 1-mm radius. Also find the value for the outer radius, keeping the inner conductor radius fixed at 1 mm is the impedance of the cable is increased to $1000\ \Omega$.

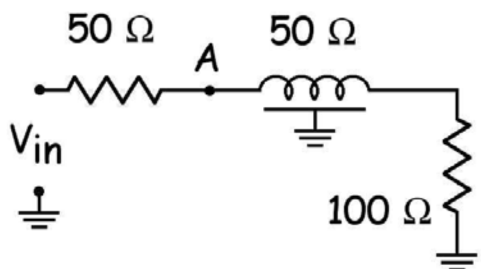
Problem 6

A 30-m long $60\ \Omega$ cable, for which the propagation velocity is *c*, has a $60\ \Omega$ resistor connected between the inner and outer conductors at one end of the cable. At the other end of the cable a 10V d.c. power source is connected across the inner and outer conductors.

(a) What is the value of the **B** field in the space between the conductors?
(a) What is the value of the **E** field in the space between the conductors?
(a) How does the direction of $\mathbf{E}\times\mathbf{B}$ depend on the polarity of the d.c. voltage source?
(a) What is the total stored energy in the electromagnetic field in the space between the two conductors?

Problem 7

The following circuit

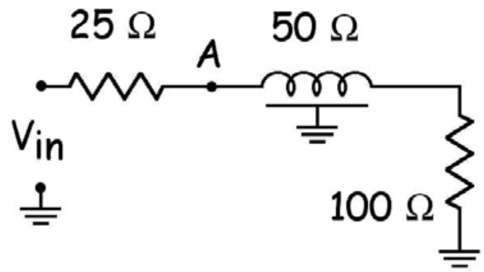


Shows an input voltage, and a $50\ \Omega$ resistor at one end of a $50\ \Omega$ delay cable with a propagation delay of 100 ns and a $100\ \Omega$ resistor at the other end. At t=0 the input voltage changes from 0V to 10V and stays at 10V.

(a) After a long time what is the voltage at point A?
(a) Plot the voltage at point A as a function of time from 0 to 1 microsecond.

Problem 8

The following circuit



Shows an input voltage, and a 25 Ω resistor at one end of a 50 Ω delay cable with a propagation delay of 100 ns and a 100 Ω resistor at the other end. At t=0 the input voltage changes from 0V to 10V and stays at 10V.

(a) After a long time what is the voltage at point A?
(b) Plot the voltage at point A as a function of time from 0 to 1 microsecond.

Problem 9

The drawing below shows a capacitor charging due to a current *I*.

(a) In the space between the plates, just at the outside edge of the capacitor (at distance *r* from the center) find the value of the Poynting vector and determine its direction.
(b) Integrate the Poynting vector over the cylindrical area between the plates and show that it is equal to the time rate of change of the energy stored in the space between the plates.