P222

Optics

Supplementary Note # 3: Mirrors

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Plane Mirrors

Let's talk about mirrors. We start with the relatively simple case of plane mirrors. Suppose we have a source of light, we will call this the *object*, located a perpendicular distance o from a plane mirror as shown in Figure 1. We consider two rays of light from the object. One ray strikes the mirror at zero angle with respect to the normal. According to the law of reflection, the reflected ray comes back along the same path. Another ray strikes the mirror at some angle θ with respect to the normal and reflects through the same angle. Now extrapolate back the two reflected rays. They meet at a point on the other side of the mirror - the rays apparently emerge from an *image*. From the drawing it is easy to see that the distance, i, between the image and mirror is the same as from the object to the mirror, or:

$$i = o$$

This is as simple as it gets. Note that the image is *virtual*. If you were to put a phototube at the image position, you would detect no photons from the source.

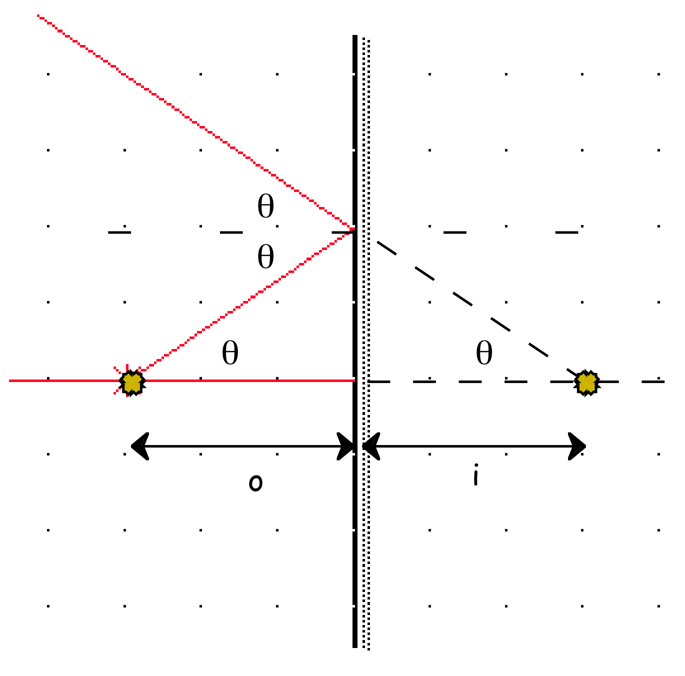

Figure 1

Looking at Yourself in a Mirror

If you look at yourself in a mirror you appear upright but left and right are reversed. If your face lacks bilateral symmetry - and we all lack that in varying degrees - your image in the mirror is different from how others see you or from a photograph of your face. Anyway, the interesting thing is that the mirror flips left with right but not top with bottom. Does this bother you?

Suppose you hold a gyroscope which is spinning along an axis which is vertical and parallel to mirror. Suppose it is spinning such that you say the angular momentum vector is pointing up. What is the direction of the angular momentum vector for the mirror image of the gyro? Now go through the same analysis but now for the axis horizontal but still parallel to the mirror plane. How are the angular momentum vectors of the object and image related. Now consider the case when the axis of spin is perpendicular to the mirror.

It is also interesting to note, as can be seen from the drawing in Figure 2, that if you want to view yourself from top to bottom, you need a mirror whose length is only half of your height.

Multiple Plane Mirrors

Consider two plane mirrors at right angles to each other as shown in Figure 3. We show a source of light and four rays emerging from the source. Rays 1 and 4 correspond to a single reflection off one mirror or the other. Again, the incident and reflected angles are symmetric about the normal to the mirror at the point where the incident ray strikes. Rays 2 and 3 emerge from the source and undergo two reflections each - one off of each mirror.

There are two trivial rays we have not drawn, one ray which is perpendicular to one of the mirrors and the other ray to the other mirror. In each case the reflected ray follows the same line. So we now consider the six reflected rays (the four drawn and the two normals) and extrapolate them back as indicated by the dotted lines to the right of and below the mirrors. The reflected rays apparently emerge from three images. Note the position of the images. The images are all virtual - again, if you put a phototube at the image positions you would not detect photons from the source. Also note that one of the images (the one which might have surprised you) is due to two reflections off of the mirrors.

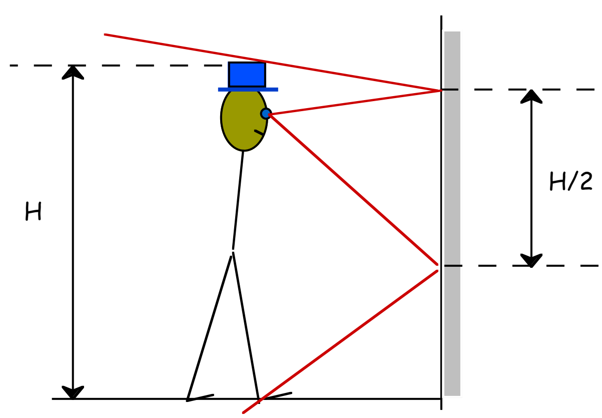

Figure 2

While we are on the subject of perpendicular mirrors, take another look at rays 2 and 3. Note that the reflected ray is parallel to the incident ray in each case. This arrangement is sometimes called a *corner reflector.* If we add a third mirror perpendicular to each of the first two, we get a really robust corner reflector.

I've drawn the object along the 45^o line bisecting the angle between the mirror. How would the picture change if the object were off to either side of the 45^o line?

It is interesting to consider what happens when you allow the angle between the mirror to vary. Calling this angle θ we just considered $\theta = \pi / 2$. The case of a single mirror corresponds to $\theta = \pi$. How many images do you expect for some arbitrary θ? It turns out that you only get a finite number of images for:

$$\theta = \frac{\pi}{N} \quad \text{N = integer}$$

and the number of images is $2N - 1$ which is in line with $N = 1$ single plane mirror with one image, $N = 2,\ \theta = \pi / 2$ for two perpendicular mirrors and $N = \infty,\ \theta = 0$ which corresponds to two parallel mirrors with a source in between. What happens for other angles?

Let's return to the case of two parallel mirrors. You get an infinite number of images. Actually you don't see an infinite number because those other images, which appear farther and farther away are due to more and more reflections. Real mirrors are not perfectly reflecting. For example, if the mirror is 90% reflective, after 10 reflections the light intensity is reduced to 35 % of the original intensity.

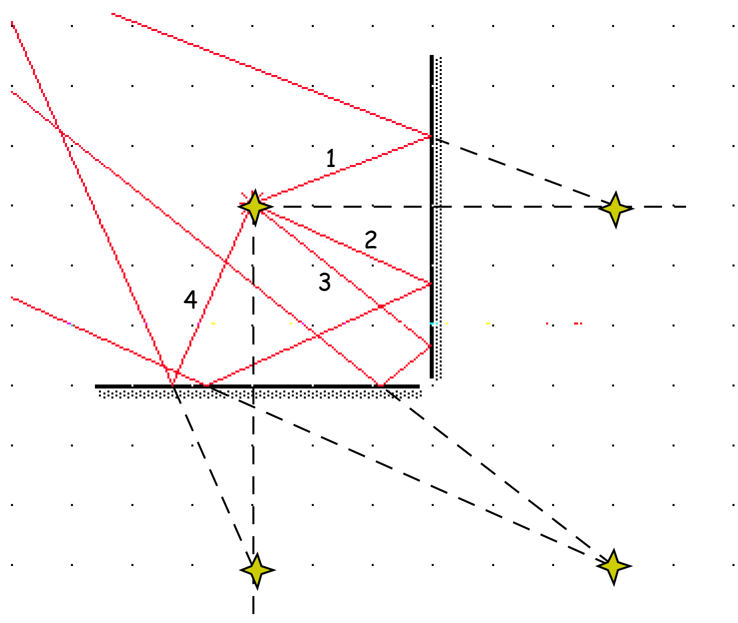

Figure 3

Spherical Mirrors

Now we will consider spherical mirrors that can either be concave or convex. Figure 4 shows a source shining on a concave mirror. The source is on the axis of the mirror. The x indicates the center of curvature. The distance between the source and mirror surface is denoted by o (object distance). Note that a ray of light along the axis would simply be reflected back along the same path. Consider a ray which emerges along a line making angle α with respect to the axis. It strikes the mirror and is reflected symmetrically with respect to the radius drawn from the center of curvature to the point on the mirror where the ray strikes the mirror. The reflected ray intersects the axis at a distance i from the mirror. This is the location of the image. We call this a *real* image. A phototube placed here would detect photons from the source. What is the relation between the distances o and i?

Referring again to the figure we note these relationships:

$$\beta = \alpha + \theta$$

$$\gamma = \alpha + 2\theta$$

From these we arrive at:

$$\alpha + \gamma = 2\beta$$

We will denote by s the arc length along the mirror from the axis to the point where the top ray strikes the mirror. We then have:

$$\alpha \approx \frac{s}{o};\ \gamma \approx \frac{s}{i};\ \beta = \frac{s}{R}$$

Note that these approximations are good only if the angles are small enough. From this we get:

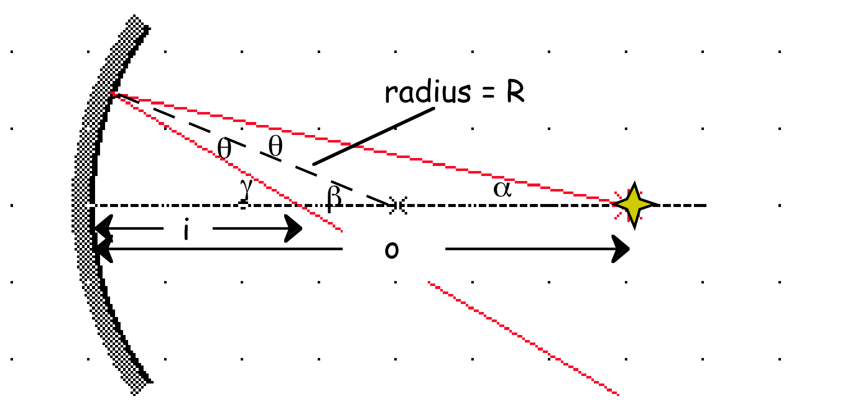

Figure 4

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

where $f = R/2$. Note that if the source is infinitely far away, then $o = \infty$ and $i = f$ - the rays are focused at the focal point.

For a plane mirror $R = \infty$ and we get $i = -o$, i.e. the object and image distances are equal. The image distance is negative, indicating that the image is on the other side of the mirror and therefore virtual.

By convention, we take the radius of curvature to be *positive* for a *concave* mirror and *negative* for a *convex* mirror.

In and earlier note we compared a parabola and circle. The paraboloid focuses perfectly and the sphere only approximately as along as the angles are small. In Figure 5 we show what happens to incident parallel rays on a concave mirror for rays which are close and far from the mirror axis. This lack of perfect focusing is referred to as a *spherical aberration*.

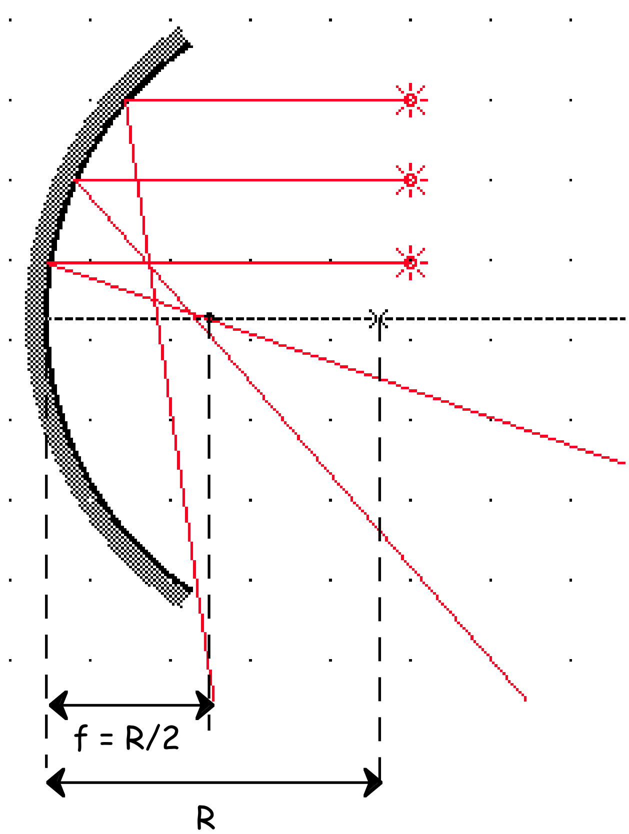

Figure 5

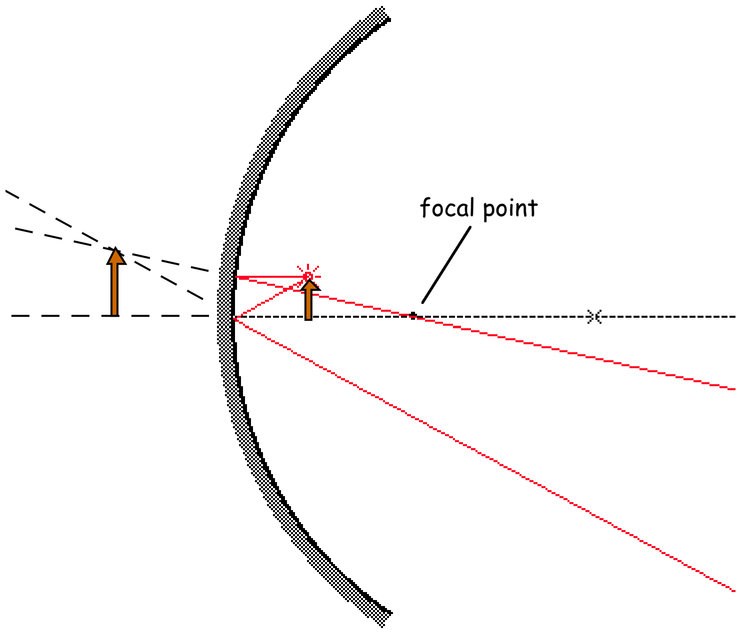

Figure 6 (a)

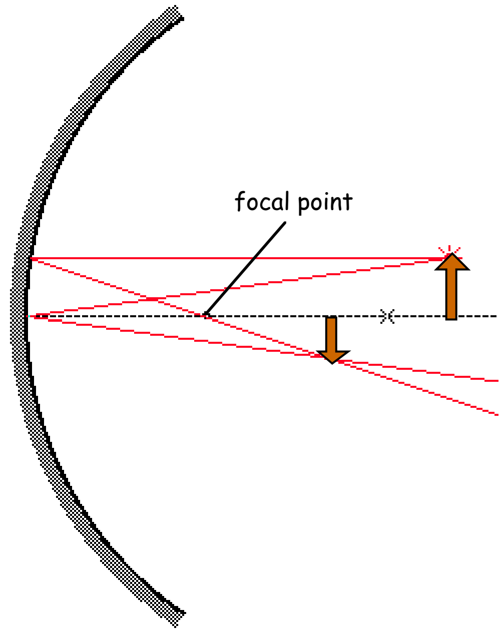

Figure 6 (b)

Concave Mirrors

Now consider Figures 6 (a) and (b). Here we show an object located in front of a concave mirror for two cases. In Figure 6 (a) the object is between the focal point and mirror and in Figure 6 (b) the object is beyond the focal point. Two rays are sufficient to find the image. Take one ray from the top of the object and parallel to the mirror axis. The reflected ray will pass through the focal point. The other ray goes from the top of the object to where the axis intersects the mirror. That ray is reflected symmetrically about the axis.

In Figure 6 (a) the extrapolated reflected rays intersect on the other side of the mirror, the image is virtual. Also, using the mirror equation above we have $i < 0$ since $o < f$, again indicating that the image is virtual. The image is upright and magnified. Magnifying mirrors used as 'makeup' mirrors have a large enough radius of curvature so that when you are reasonably close to the mirror you see an upright and magnified image of your face. The background behind you is, however, inverted. More on that now.

Figure 6 (b) shows the reflection of the parallel ray and the ray incident from the top of the object to the center point on the mirror. The reflected rays now intersect on the same side of the mirror as the object but now the object is inverted. So the image is real and inverted. Inspection of the figure also shows that the magnification is given by:

$$m = -\frac{i}{o}$$

Note the minus sign which indicates that the image is inverted. Note that if the object is at the center of curvature the image and object coincide and the image is inverted and has the same size as the object.

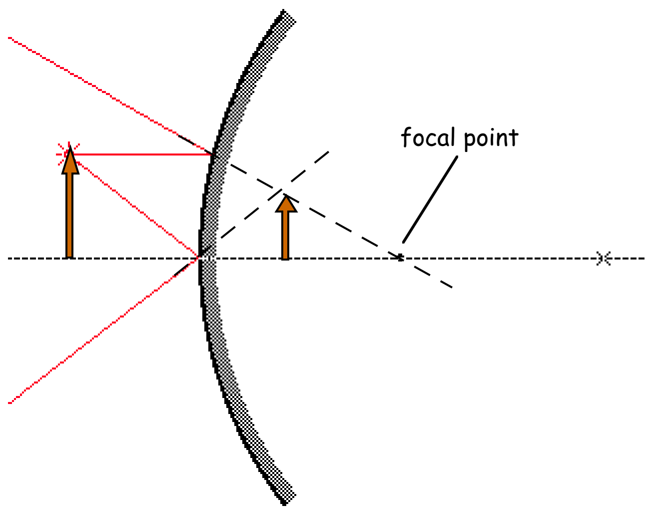

Figure 7

Convex Mirror

Figure 7 shows the two reference rays for an object placed in front of a convex mirror. The focal length is negative which means then that the image distance is always negative (image on the other side of the mirror and virtual) and the image is upright (positive magnification). The image is always smaller than the object as well. The passenger-side side view mirror on most cars is a convex lens with the warning that "objects in the mirror are closer than they appear" as you will remember from the movie *Jurassic Park*.

I generated the ray tracing diagrams in this note with the Ray program distributed by Physics Acadmic Software. This program is available for your use.