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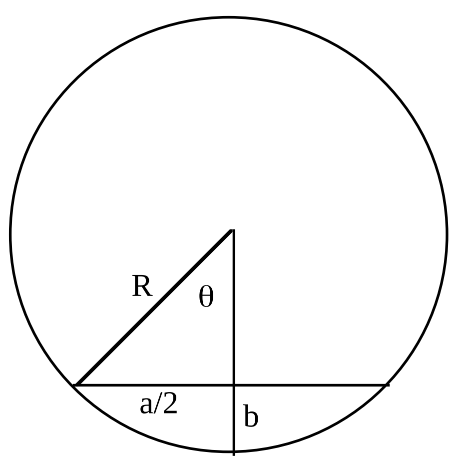
Measuring the Focal Length of a Lens
An Exercise in Error Analysis

Using a Spherometer

This is a report on measuring two lens, a plano-convex and a plano-concave lens using a spherometer. This device has three fixed legs arranged on the vertices of an equilateral triangle. A screw positions (up or down) a fourth leg at the center of the equilateral triangle. The screw drive has a disc the height of which can be read off in mm. The disc also has a vernier scale. Two full rotations of the screw corresponds to movement of the center leg by 1 mm. This was verified by measuring the number of threads per mm. For the spherometer used, the points of the equilateral triangle lie on a 32-mm diameter circle. Thus the spherometer measures the sagitta (b) as shown in the figure below. From this one can compute the radius. One calibrates the spherometer by placing it on a flat surface and adjusting the middle leg so that it just touches the surface.

From the figure we see that $b = R \cdot (1 - \cos\theta)$ and that $a = 2 \cdot R \cdot \sin\theta$ and from this:

$$R = \frac{b^2 + a^2/4}{2b}$$



We measured the radius of curvature for the converging lens and for the diverging lens and measured $b = 1.25 \pm 0.13$ mm and $b = 2.25 \pm 0.13$ respectively. The respective error in R is calculated as follows:

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial a} \cdot \delta a\right)^2 + \left(\frac{\partial R}{\partial b} \cdot \delta b\right)^2}$$

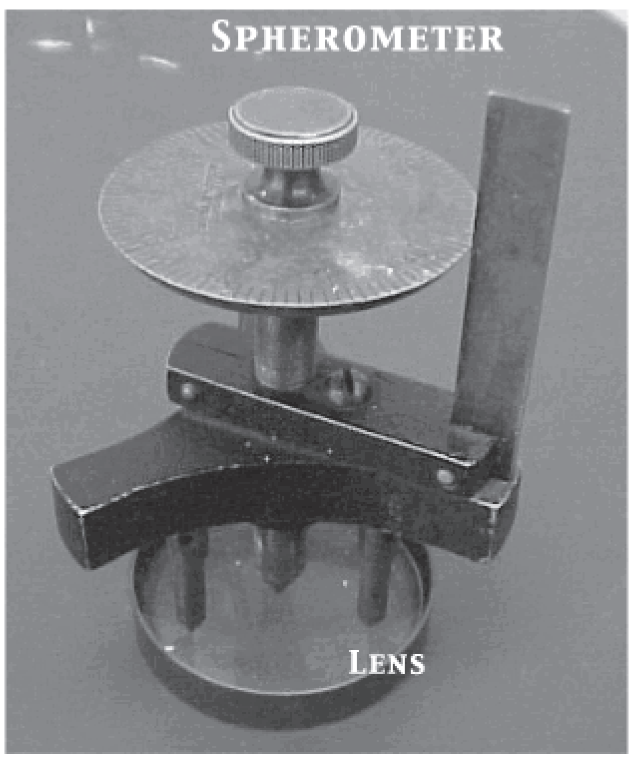
$$\frac{\partial R}{\partial a} = \frac{a}{4b}$$

$$\frac{\partial R}{\partial b} = \left(1 - \frac{R}{b}\right)$$

From this we have: R(converging) = 103.0 ± 10.3 mm and R(diverging) = 61.3 ± 3.8 mm. To translate this into focal length, use:

$$f = \frac{R}{1 - n} = \frac{R}{0.52}$$

from which: f(converging) = 19.8 ± 2.0 cm and f(diverging) = 11.8 ± 0.7 cm.



Using a Lens Gauge

This device has a dial readout and three short prongs - the middle one is spring driven and adjusts to touch the surface of a lens. The readout is in diopters. A diopter is just the inverse of the focal length expressed in meters. Converting our above measurements to diopters yields 5.1 ± 0.5 D for the converging lens and 8.5 ± 0.5 D for the diverging lens. Using the lens gauge we measure 5.00 ± 0.03 D for the converging lens and 8.20 ± 0.05 D for the diverging lens. These translate into focal lengths of 20 cm and 12 cm respectively.

The lens gauge is much easier to use.

Using the Lens Gauge to Measure the Focal Length of the Mirage Mirror

I measured 2.8 D using the lens gauge. This converts into R by correcting for 1-n, thus:

$$R(\text{meter}) = \frac{0.52}{D} = 0.17$$

so the focal length is about 8.5 cm.