LR Circuits

Honors Physics Note 003
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Honors Physics P222 - Spring, 2004

Introduction

This note concerns the behavior of circuits that include combinations of resistors, inductors and possibly a battery.

Inductors

Suppose you wind an insulated wire tightly around a pencil and then remove the pencil and attach the ends of the wire to a voltage source. A current will flow through wire and set up a magnetic field inside space enclosed by the wire winding (this particular arrangement is called a *solenoid*). If you try to change the current, a reverse voltage is induced to oppose the change – this is a consequence of Faraday's law, as we will see when we study Maxwell's equations. The induced voltage across the entire winding is proportional to the rate of change of current and the proportionality factor is called the inductance L:

$$V = L\frac{dI}{dt} \tag{1}$$

As with capacitance, the inductance depends only on geometry and the material inside the current winding (we will assume vacuum). In this course we will calculate the inductance for different geometries. Similar to capacitance, one way to calculate the inductance is to integrate the energy density stored in the magnetic field over the volume of the inductor and that energy density is given by $B^2/2\mu_o$ where B is the magnetic field and is proportional to current I and μ_o is the *magnetic permeability* of free space and in SI units is $4\pi \times 10^{-7}\ N/A^2$. In analogy to capacitors, the energy stored in an inductor carrying current I is:

$$U = \frac{1}{2}LI^2 \tag{2}$$

The unit for inductance is the henry H in units where current is measured in A, voltage in volts and time in s.

Series and parallel combinations of inductors

The rules for combining series and and parallel combinations of inductors are the same as for resistors. A series combination of L_1 and L_2 can be replaced by its equivalent L_{eq} where:

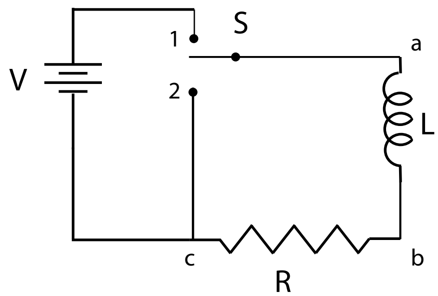

Figure 1: A circuit with a resistor R and inductor L in series along with a battery V and a switch S. When the switch is closed in position 1 the resistor and conductor are in series across the battery and current flows from point a to b to c. But the current does not reach its final value instantaneously – it builds up to it final value. While the current is changing the voltage at a is higher than the voltage at b across the inductor. Eventually the current reaches the value V/R and stays constant at which point there is no voltage drop across the inductor. When the switch is then moved to position 2 (after a long time in position 1) the current does not drop to zero instantaneously. The voltage drop across the inductor has magnitude LdI/dt and accounts for the current across the resistor. Eventually the current drops to zero.

$$L_{eq} = L_1 + L_2 \tag{3}$$

and a parallel combination of L_1 and L_2 can be replaced by its equivalent L_{eq} where:

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \tag{4}$$

Circuits with inductors and resistors

Figure 1 shows a circuit with a resistor R and inductor L in series along with a battery V and a switch S. When the switch is closed in position 1 the resistor and conductor are in series across the battery and current flows from point a to b to c. But the current does not reach its final value instantaneously – it builds up to it final value. While the current is changing the voltage at a is higher than the voltage at b across the inductor. Eventually the current reaches the value V/R and stays constant at which point there is no voltage drop across the inductor.

Applying the Kirchoff law for the sum of the voltages around a closed loop we have:

$$V - L\frac{dI}{dt} - RI = 0 \tag{5}$$

and it can be verified that the solution to equation 5 is:

$$I(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \tag{6}$$

Indeed the current is initially zero and eventually reaches the value V/R. The quantity L/R must have units of time and defines the time constant for the circuit – the measure of time required for the current to build up to its final value.

When the switch is then moved to position 2 (after a long time in position 1) the current does not drop to zero instantaneously. The voltage drop across the inductor has magnitude LdI/dt and accounts for the current across the resistor. Eventually the current drops to zero.

Equation 5 is now replaced by:

$$L\frac{dI}{dt} + RI = 0 \tag{7}$$

The solution to equation 7 is:

$$I(t) = I_0 e^{-Rt/L} \tag{8}$$

Problems

Problem 1

Show that the rules for combining series and parallel combinations of two inductors are indeed given by equations 3 and 4 respectively.

Problem 2

Verify that equation 6 is indeed the solution to equation 5.

Problem 3

Equation 6 describes the build up of current in the circuit of Figure 1 assuming the switch is closed in position 1 at $t = 0$. In units of the time constant $\tau = L/R$, how much time is required for the current to build up to 50% of its final value and up to 90% of its final value?

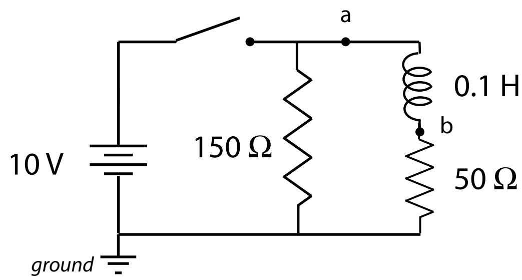

Figure 2: Problem 4

Problem 4

Figure 2 shows a 10 V battery in a circuit. Assume that the switch S has been closed for a long time.

(a) While the switch is closed, that are the currents in the 150 Ω resistor, the 50 Ω resistor and the 0.1 H inductor?

(b) While the switch is closed what is the voltage at point b with respect to ground?

At $t = 0$ the switch is open.

(c) Make a plot of the potential at point a with respect to ground for $t < 0$ and several tens of milliseconds after the switch is opened.

(d) Make a plot of the potential at point b with respect to ground for $t < 0$ and several tens of milliseconds after the switch is opened.